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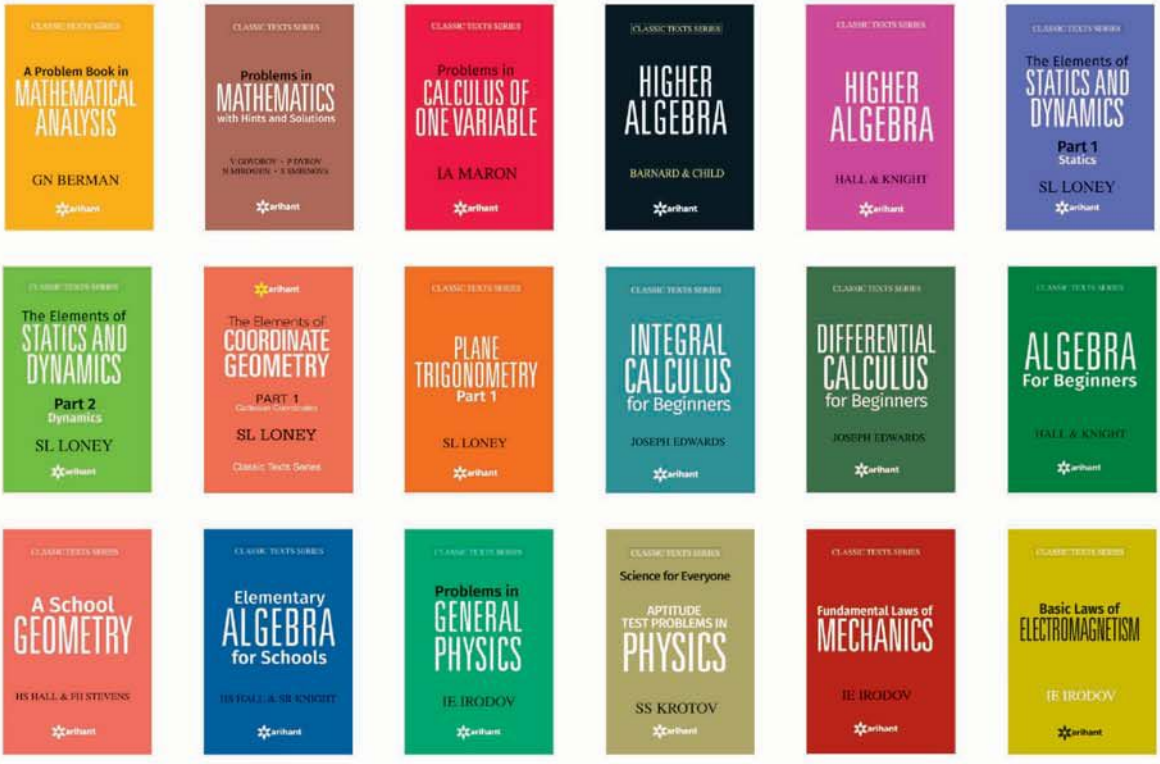

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CONTENTS

02 FROM THE EDITOR'S DESK
THE STUMBLING BLOCKS

04 TEST TUNER
JEE Advanced Prep Up

14 TEST TUNER
JEE Advanced Prep Up

26 TEST TUNER
JEE Advanced Scale Up

30 SHORTCUT METHODS

36 OLYMPIAD PRACTICE

38 XII CBSE BOARD PAPER 2016

48 CONCEPT MAP

50 WBJEE 2016
Practice Paper

55 BRAIN TEASERS

58 TEST TUNER
BITSAT Prep Up

66 TEST TUNER
BITSAT Prep Up

74 SPECTRUM FOR YOU

76 TEST TUNER
BITSAT Scale Up

80 JEE MAIN 2016
Solved Paper

89 JEE ADVANCED PRACTICE
Periodicity of Functions

95 PARTICIPATE & WIN

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PARUL JAIN

Chairman
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Editorial Head
SANJAY SHARMA

Publishing Manager
AMIT VERMA

Coordinator
ZEBA SIDDIQUE

Production Manager
SHISHPAL SINGH

Circulation & Advertisement General Manager
ANIL KUMAR GUPTA
+91-9219619948
anilgupta.arihant@gmail.com

Cover & Layout Design
SHANU MANSOORI

Page Designer
AMIT BANSAL

Diagram
BRAHAMPAL SINGH

Type Setting
MAYANK, SANDEEP, ASHISH

Proof Reader
DINESH

Copy Editor
PRACHI

Head Office
ARIHANT MEDIA PROMOTERS
KALINDI, TP NAGAR, MEERUT-2

Phone 0121-2401479, 2512970
Fax 0121-2401648
E-mail spectrum@arihantbooks.com
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THE STUMBLING BLOCKS

All of us are/were television, internet or computer games addicts at some point of times in our lives. It is up to us to convert this kind of addiction into distraction or a utility e.g., I was a TV addict during early days of my life, but today it is more or less a utility for me. I use TV, Movies etc., for recreation purposes when I am mentally tired. Infact these two elements (TV shows and Movies) work as energy boosters for my body.

Scientifically frames in a movie or a TV show change faster that the functioning rate of our brain. That's why our real thinking power decreases up to a large extent while watching these. Actually our brain works only over those directions reaching to it *via* our eyes while watching a movie or a TV show. Thus, keeping other portions of our brain at minimum functional level, this in turn provides relief up to a large extent to our tired functional brain. On the other hand, watching TV and Movies may affect our body in negative manner also. Their over exposure is always hazardous to our eyes and other body parts.

The journey to IIT is too long. It seems like walking on the top of a moving object which is rough, slippery and narrow too. Walking on such an object's top requires ultimate skills, courage, consistency and determination as key players. One 'slip' can easily deviate you from the track. The word **'slip'** here signifies **'Distraction'**. There will be all kinds of things like *TV, movies, internet, phone, friends, 'Bad' habits* (smoking, drinking etc.,), *crushes/infatuations, the pressure of looking good, the desire of being noticed in social circle* etc., which can be categorised to distractions or stumbling block for your progress. Sometimes, a mode of recreation also becomes a distraction.

THE REAL STUMBLING BLOCKS

From the example given above, it must be clear in your mind *that controlled exposure to gadgets like TV, internet etc., may not distract you, but uncontrolled exposure certainly distract you just like a hydrogen bomb which demolished your studies completely*. Hence, I strongly suggest every JEE aspirant to analyse and remove from his path such stumbling blocks which can be one of the following or something else

TELEVISION

Let's begin with TV, 'DOORDARSHAN' which have the capability of converting a normal human being into 'Buddhu' or an 'idiot'. That's why some of us also call it 'Buddhu Baksa' or 'idiot box'. For one kind of people this can be the source of entertainment through reality shows, serials etc., while for others it provides entertainment through games like IPL, IHL, FIFA, Grand slams, ICC world cups etc.

Thus, something or the other always visible on the screen and pulling you away from your books just like spider webs. For a person like my son, it was even a bigger problem because he (and many more like him) equally fits in both the categories. Likewise his younger cousin says that he used to be glued to the television every time Virat stepped out to bat, just like a normal cricket fan. First watching a match and then the guilt of wasting time is a great distraction for him.

To overcome this kind of problem, you really need to have burning desire to hit your target. Once you are involved with such attractions, it will be hard to come back. Suppose you decided to sit for an hour in front of the TV and ended up watching a 3 hour T20 match. This in turn followed by wasting another hour in discussing the outcomes of match or feeling guilty.

Everybody understands the fact that it is almost impossible to be passive emotionally during this tenure of 2 years. There will be ups and downs signifying good and bad times during this tenure. These up and downs are in fact the 'Testing Times' for us. *I consider these times as the eliminators as these are the indicators of our success and failures*. What would give you more happiness – *a **good rank in JEE** or India lifting the World Cup 2015 in cricket?* Think about this question and your heart would give good rank in JEE as the answer.

You can, or rather should, surely watch programs of your interest but it is better to fix up a time for TV and try to follow it honestly. Be stiff with yourself. Build the road map of your priorities in such a way to get maximum help from it in ensuring your success.

INTERNET

Internet entered in our life as a time saving connecting tool which provides manifold help in increasing our exposure towards the real world. Although internet has something for everybody, but amazingly everyone among us is using it for the same purpose i.e. Social Networking means everybody is running towards social networking through many apps like Facebook, Twitter, Instagram etc., including you, your parents, your siblings and even your pets may have an account. Your worth is judged by the number of friends or followers you have. You are called COOL, if you have a good collection of songs or movies else you stand nowhere. For most people, social networking is not a requirement, but just an addiction.

I realised it when I lost my account for a month due to some reason. In real sense, when you don't have anything to post, you don't even expect a comment also. Just try to deactivate your account once, then only you will realise the worth of life without such social networking. In real sense it is much – much better than the one you are living with Facebook or Instagram. In a very recent research, this was disclosed that addiction of this kind of social media is more prone in those persons, which have some sort of insecurities in their relations. The solution of these kinds of distractions lies in your determination and commitment for success. So, *I prefer the usage of internet could be the best, if students use it for*

- The purpose of clearing their study related doubts
- For online tutorials/classes
- For reading e-books
- For exploring new horizons in different fields etc.

Likewise, if their parents use it for

- Office automation,
- Accounting purposes,
- Financial operations like bank accounts,
- Online shopping i.e., e–commerce, and
- So many other useful things.

MOBILE PHONE

I consider it a dragon that always stays in your pocket and literally grabs the peace of your life. I always prefer its limited usage as it left me unnecessarily exhausted. It is suggestive to switch off your mobiles when you are studying. Try to avoid its use in your 'relaxation time' too i.e., your **own time** in which

- You can have a chat with your family members or
- You can play some outdoor games, or
- You can take a walk in the park to rejuvenate your exhausted body
- You can listen to some music to re-energize yourself.

Simply, this means the time of your personal usage, which must be used for constructive purposes. Some of you may be of the opinion that the usage of mobile phone is necessary to increase and maintain their social circle. But as per my understanding an increase in 'social circle' does not necessarily means an increase in number of 'friends'. Likewise, its size has inverse relation with your success.

REMEMBER!

The day you compromise with your success for them, they will compromise on their 'friendship' for you. In simple words, they are not your true friends rather they want to gain something from your success. Hence, when you make compromises with your success they may leave you. So limit your social circle only to the people, who know the worth of your time.

CRUSHES AND INFATUATIONS

Crushes and infatuations are the part of every adolescent's life. Attraction to opposite sex at this age is very common, but at the same time its proper incorporation in life is essential. At such times, when each minute has its own value and requires proper planning, don't even think of having a girlfriend or a boyfriend for time pass. Their presence is acceptable only if, they do not deviate you from what you are supposed to do? Crushes may keep on changing every month i.e., you can fall in love 24 times or more in these 2 years, but you won't allowed to appear 24 times to crack JEE. *So you have two options i.e., either learn to handle both the things simultaneously or become a saint until you crack the JEE.*

In my opinion the second option is better as at your age nobody is mature enough to handle both the things simultaneously.

ATTENTION SEEKERS

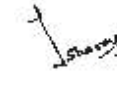
Seeking attention may also be considered as a distraction. See, if looking good makes you feel 'confident', then it is fine, but if it makes you feel 'acceptable', then it is wrong. Try to please everyone with your result and not with your so called personality. My perception is, in your age personality keeps on changing in days or weeks. Accept yourself as a future star, who can do much more than the rest without following their ways. Love yourself and respect your uniqueness. Let people judge you, but what you think about yourself matters the most. Just decide what appeals you the most - a 'show man' or a 'show stopper'.

All the above written distractions are those common factors which everyone among us faces or faced in his/her life. You are not alone the only victim of these.

REMEMBER!

For clearing JEE, you need not at all isolate yourself from your friends, family or any modes of recreation. Rather, it is very necessary to give them the time they deserve. **Play for half an hour or talk to your parents or play with your siblings or watch TV, chat, dance, sing, sketch** etc., in short, do whatever, which makes you forget that you are up for a job, but with a restriction. In the early days of my career, I had a student who was very fond of cricket. He got his selection to IIT with a marvelous rank of 325. Just imagine, he was playing a cricket match 4 days before his IIT-JEE, in which he got this rank. This was a lesson for me regarding distractions. If we analyse this incident properly, we can easily understand the difference between stumbling blocks and utilisation of above written distractions with focused approach. That's why I mentioned above, do whatever you like, but be focused in your approach. See how beautifully **Abraham Lincoln** connected these distractions with destiny.

"'Always bear in mind that your resolution to succeed is more important than any other one thing."



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Paper I

Section 1

Single Digit Integer Type

1. If $f(x)$ and $g(x)$ are two polynomials such that $h(x)=xf(x^3)+x^2g(x^6)$ is divisible by x^2+x+1, then the value of $g(1)$ is

2. Let $f(x)=x^2-1$ and

$$g(x)=\begin{cases}[|f(|x|)|]+|f(x)|; & x\in(-1,0)\cup(0,1)\\ 1 \quad ; & \text{Otherwise}\end{cases}$$

Then, the number of elements in the range of $[|g(x)|]$ is (where, $[\cdot]$ denotes the greatest integer function)

3. If $\alpha,\beta;\beta,\gamma;\gamma,\alpha$ are the roots of $a_rx^2+b_rx+c_r=0$ for $r=1,2,3$, where $\alpha,\beta,\gamma>0$

and $\Sigma\alpha+\Sigma\alpha\beta+\alpha\beta\gamma=\left[\prod_{r=1}^{3}\left(\frac{a_r-b_r+c_r}{a_r}\right)\right]^{1/2}-k$, then

k is equal to

4. The sum of first n terms of the sequence $1,1+2,(1+2+2^2),\ldots,(1+2+2^2+\ldots+2^{n-1})$ is of the form $2^{n+A}+Bn^2+Cn+D,\ \forall\ n\in N$.

The value of $|A+B+C+D|$ is

5. If the integral solution of the equation

$(x-1)^3+(x-2)^3+(x-3)^3+\ldots+(x-2006)^3+(x-2007)^3=0$

is only k, then $\left(\frac{k}{502}\right)$ is equal to

6. The number of solutions of the equation $|\sin x\cdot\cos x|+\sqrt{2+\tan^2 x+\cot^2 x}=\sqrt{3}$;

$x\in(0,6\pi)-\left\{n\pi,(2n+1)\frac{\pi}{2}\right\}$, where $n\in I$, is

7. The infinite geometric series $\sum_{r=0}^{\infty}\frac{a}{(y^2-4y+5)^{r-1}}$;

where $y=x^2-6x+11$; has a finite sum, then x cannot be equal to

8. The equation to the side BC of a ΔABC is $x=2$. If altitude through A meets the circumcircle of ΔABC at $P(7,-2)$, then the absolute value of the sum of the x and y-coordinates of the orthocentre of ΔABC is equal to

Section 2

One or More than One Options Correct Type

9. If (α,α^2) lies inside the triangle formed by the lines $2x+3y-1=0$, $x+2y-3=0$ and $5x-6y-1=0$, then

(a) $2\alpha+3\alpha^2-1>0$ (b) $\alpha+2\alpha^2-3>0$

(c) $\alpha+2\alpha^2-3<0$ (d) $6\alpha^2-5\alpha+1>0$

10. Let $A(k)$ be the area bounded by the curves $y=x^2-3$ and $y=kx+2$.

(a) The range of $A(k)$ is $\left[\frac{10\sqrt{5}}{3},\infty\right)$

(b) The range of $A(k)$ is $\left[\frac{20\sqrt{5}}{3},\infty\right)$

(c) If function $k\to A(k)$ is defined for $k\in[-2,\infty)$, then $A(k)$ is many-one function.

(d) The value of k for which area is minimum is 1.

11. If $f(x)=\min\{x,x^2\}$, then

(a) $f(x)$ is continuous, $\forall x\in R$

(b) $f'(x)=1,\ \forall\ x>1$

(c) $f(x)$ is differentiable, $\forall\ x\in R$

(d) $f(x)$ is not differentiable at two points

12. Which of the following function has point of extrema at x = 0 ?

(a) $f(x) = e^{-|x|}$

(b) $f(x) = \sin|x|$

(c) $f(x) = \begin{cases} x^2 + 4x + 3 & ; \quad x < 0 \\ -x & ; \quad x \geq 0 \end{cases}$

(d) $f(x) = \begin{cases} |x| & ; \quad x < 0 \\ \{x\} & ; \quad 0 \leq x < 1 \end{cases}$

(where, $\{x\}$ represents fractional part function)

13. If $f(x) = \begin{cases} x^2\,(\text{sgn}([x]) + \{x\}) & ; \quad 0 \leq x < 2 \\ \sin x + |x - 3| & ; \quad 2 \leq x < 4 \end{cases}$

where, $[\cdot]$ and $\{\cdot\}$ represent the greatest integer and the fractional part function, respectively

(a) $f(x)$ is differentiable at $x = 1$

(b) $f(x)$ is continuous but non-differentiable at $x = 1$

(c) $f(x)$ is non-differentiable at $x = 2$

(d) $f(x)$ is discontinuous at $x = 2$

14. $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are unimodular and coplanar. A unit vector $\mathbf{d}$ is perpendicular to them. If $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = \frac{1}{6}\hat{\mathbf{i}} - \frac{1}{3}\hat{\mathbf{j}} + \frac{1}{3}\hat{\mathbf{k}}$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is 30°, then $\mathbf{c}$ is equal to

(a) $\frac{1}{3}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ *(b)* $\frac{1}{3}(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

(c) $\frac{1}{3}(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$ *(d)* $\frac{1}{3}(-2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$

15. If $z_1 = 5 + 12i$ and $|z_2| = 4$, then

(a) $\max(|z_1 + iz_2|) = 17$

(b) $\min(|z_1 + (1 + i)z_2|) = 13 - 4\sqrt{2}$

(c) $\min\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{4}$

(d) $\max\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3}$

16. Probability if n heads in $2n$ tosses of a fair coin can be given by

(a) $\prod_{r=1}^{n} \frac{2r-1}{2r}$ *(b)* $\prod_{r=1}^{n} \frac{n+r}{2r}$ *(c)* $\sum_{r=0}^{n} \left(\frac{{}^nC_r}{2^n}\right)^2$ *(d)* $\frac{\sum_{r=0}^{n} ({}^nC_r)^2}{\sum_{r=0}^{2n} {}^{2n}C_r}$

17. If the foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ coincide with the foci of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and eccentricity of the hyperbola is 2, then

(a) $a^2 + b^2 = 16$

(b) there is no director circle to the hyperbola

(c) centre of the director circle is (0, 0)

(d) length of latusrectum of the hyperbola = 12

18. If $\int \frac{e^{x-1}}{(x^2 - 5x + 4)} 2x\,dx = Af(x-1) + Bf(x-4) + C$ and $f(x) = \int \frac{e^x}{x} dx$, then

(a) $A = -\frac{2}{3}$ *(b)* $B = \frac{4}{3}e^3$

(c) $A = \frac{2}{3}$ *(d)* $B = \frac{8}{3}e^3$

Section 3

Match the Columns Type

19. Match the statements of Column I with the values of Column II.

	Column I		Column II
A.	$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-5}{5}$ are	p.	coincident
B.	$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-3}{2} = \frac{y-5}{3} = \frac{z-7}{4}$ are	q.	parallel and different
C.	$\frac{x-2}{5} = \frac{y+3}{4} = \frac{5-z}{2}$ and $\frac{x-7}{5} = \frac{y-1}{4} = \frac{z-2}{-2}$ are	r.	skew
D.	$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z-4}{5}$ and $\frac{x-3}{3} = \frac{y-2}{2} = \frac{z-7}{5}$ are	s.	intersecting in a point
		t.	coplanar

20. Match the statements of Column I with the values of Column II.

	Column I	Column II
A.	The number of integral points (integral point means both the coordinates should be integer) that lie exactly in the interior of the triangle with vertices (0,0), (0, 21), (21,0) is	p. greater than 50
B.	If 7 points out of 12 are in the same straight line, then the number of triangles formed is	q. greater than 100
C.	The number of subsets of the set {1, 2, 3, ..., 9} containing atleast one odd number, is	r. greater than 150
D.	The number of proper divisors of 38808 is	s. greater than 200
		t. greater than 300

Paper II

Section 1

Single Digit Integer Type

1. If $\frac{\log x}{2} = \frac{3}{\log y} = \frac{\log z}{5}$ and $zx = 10$, then the value of $\log\left(\frac{y}{z^{14}x^{14}}\right)$ is (base of log is 10)

2. In a ΔABC, if $\tan A = 1/2$, $\tan B = k + 1/2$ and $\tan C = 2k + 1/2$, then the possible value of $[k]$, where $[\cdot]$ denotes the greatest integer function, is

3. If $\mathbf{x}$, $\mathbf{y}$ are two non-zero and non-collinear vectors satisfying $[(a-2)\alpha^2 + (b-3)\alpha + c]\mathbf{x} + [(a-2)\beta^2 + (b-3)\beta + c]\mathbf{y} + [(a-2)\gamma^2 + (b-3)\gamma + c](\mathbf{x}\times\mathbf{y}) = 0$, where α, β, γ are three distinct real numbers, then the value of $a^2 + b^2 + c^2 - 4$ is

4. If the length of the projection of the line segment with points $(1, 0, -1)$ and $(-1, 2, 2)$ to the plane $x + 3y - 5z = 6$ is d, then the value of $\left[\frac{d}{2}\right]$, where $[\cdot]$ represents the greatest integer function, is

5. The solution set of inequality $(\cot^{-1} x)(\tan^{-1} x) + \left(2 - \frac{\pi}{2}\right)\cot^{-1} x - 3\tan^{-1} x - 3\left(2 - \frac{\pi}{2}\right) > 0$ is (a, b), then the value of $\cot^{-1} a + \cot^{-1} b$ is

6. A circle inscribed in a ΔABC touches the side AB at D such that $AD = 5$ and $BD = 3$. If $\angle A = 60°$, then the value of $\left[\frac{BC}{3}\right]$ is (where, $[\cdot]$ denotes the greatest integer function)

7. If the set of values of 'a' for which $f(x) = ax^2 - (3 + 2a)x + 6$; $a \neq 0$ is positive for exactly three distinct negative integral values of x is $(c, d]$, then the value of $(c^2 + 4|d|)$ is equal to

8. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+4} \\ y^n & y^{n+2} & y^{n+4} \\ z^n & z^{n+2} & z^{n+4} \end{vmatrix} = \left(\frac{1}{y^2} - \frac{1}{x^2}\right)\left(\frac{1}{z^2} - \frac{1}{y^2}\right)\left(\frac{1}{x^2} - \frac{1}{z^2}\right)$, then $-n$ is equal to

Section 2

One or More than One Options Correct Type

9. If $f(m) = \sum_{i=0}^{m} ({}^{30}C_{30-i})({}^{20}C_{m-i})$, then

(a) maximum value of $f(m)$ is ${}^{50}C_{25}$

(b) $f(0) + f(1) + \ldots + f(50) = 2^{50}$

(c) $f(m)$ is always divisible by $50 (1 \le m \le 49)$

(d) the value of $\sum_{m=0}^{50} [f(m)]^2 = {}^{100}C_{50}$

10. Three sides of a triangle have the equations $L_i \equiv y - m_i x = 0; i = 1,2,3.$ The expression $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$, where $\lambda \neq 0, \mu \neq 0$, is the equation of the circumcircle of the triangle, if

(a) $1 + \lambda + \mu = m_1m_2 + \lambda m_2m_3 + \mu m_3m_1$

(b) $m_1(1 + \mu) + m_2(1 + \lambda) + m_3(\mu + \lambda) = 0$

(c) $\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} = 1 + \lambda + \mu$

(d) None of the above

11. Consider $f(x) = \cos^{-1} x + \cos^{-1}\left(\frac{\sqrt{3}x + \sqrt{1 - x^2}}{2}\right)$.

If $\sum_{r=1}^{100} f\left[1 - \left(\frac{1}{10}\right)^r\right] = \frac{p}{q}\pi$, where p and q are relatively prime, then

(a) $p - 16q = 2$

(b) $\left[\frac{p}{q}\right] = 16$ (where, $[\cdot]$ denotes GIF)

(c) $\frac{p + 16q}{2} = 49$

(d) $\left[\frac{p}{q}\right] = 18$ (where, $[\cdot]$ denotes the GIF)

12. Radhika has 10 friends among whom two are married to each other. She wished to invite five of them for a party. If the married couples refuse to attend separately, then the number of different ways in which she can invite five friends, is

(a) ${}^8C_5$ (b) $2 \times {}^8C_3$

(c) ${}^{10}C_5 - 2 \times {}^8C_4$ (d) None of these

13. A normal drawn to a parabola $y^2 = 4ax$ at a point P, meet the curve again at Q such that angle subtended by PQ at vertex is $90°$, then the coordinates of P can be

(a) $(8a, 4\sqrt{2}a)$ (b) $(8a, 4a)$

(c) $(2a, -2\sqrt{2}a)$ (d) $(2a, 2\sqrt{2}a)$

14. Let $f(x) = \frac{3}{4}x + 1$ and $f^n(x)$ be defined as $f^2(x) = f(f(x))$ and for $n \ge 2, f^{n+1}(x) = f(f^n(x))$. If $\lambda = \lim_{n\to\infty} f^n(x)$, then

(a) λ is independent of x

(b) λ is linear polynomial in x

(c) the line $y = \lambda$ has slope 0

(d) the line $4y = \lambda$ touches the unit circle with centre at the origin

15. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is equal to

(a) $f_n(x)\frac{d}{dx}\{f_{n-1}(x)\}$ (b) $f_n(x)f_{n-1}(x)$

(c) $f_n(x) f_{n-1}(x) \ldots f_2(x) \cdot f_1(x)$ (d) None of these

16. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is

(a) 2 (b) -2 (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

Section 3

Linked Comprehension Type

Passage 1

Directions (Q. Nos. 17-18) *If the squares of* 8×8 *chessboard are painted either red or black at random.*

17. The probability that not all the squares in any column are alternating in colour is

(a) $\left(1 - \frac{1}{2^7}\right)^8$ (b) $\frac{1}{2^{56}}$

(c) $1 - \frac{1}{2^7}$ (d) None of these

18. The probability that the chessboard contains equal number of red and black squares, is

(a) $\frac{{}^{64}C_{32}}{2^{64}}$ (b) $\frac{64!}{322^{64}!}$

(c) $\frac{2^{32} - 1}{2^{64}}$ (d) None of these

Passage 2

Directions (Q. Nos. 19-20) *Let* $\mathbf{r}$ *be a position vector of a variable point in XY-plane, such that*

$\mathbf{r} \cdot (10\hat{\mathbf{j}} - 8\hat{\mathbf{i}} - \mathbf{r}) = 40$ *and*

$p_1 = \max \{ | \mathbf{r} + 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}|^2 \}$,

$p_2 = \min \{ | \mathbf{r} + 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}|^2 \}$

A tangent line is drawn to the curve $y = \frac{8}{x^2}$ *at the point A with abscissa* 2. *The drawn line cuts the X-axis at a point B.*

19. p_2 is equal to

(a) 9 (b) $2\sqrt{2} - 1$

(c) $6\sqrt{2} + 3$ (d) $9 - 4\sqrt{2}$

20. $\mathbf{AB} \cdot \mathbf{OB}$ is equal to

(a) 1 (b) 2

(c) 3 (d) 4

Answers *with* Explanation

Paper I

1. ***(0)*** We have, $h(x)$ is divisible by $x^2 + x + 1$ and $x^2 + x + 1 = 0$ have roots ω and ω^2, therefore $h(\omega)$ and $h(\omega^2)$ must be zero, i.e.

$$h(\omega) = \omega f(\omega^3) + \omega^2 g(\omega^6) = \omega f(1) + \omega^2 g(1) = 0 \quad \ldots(i)$$

and $\quad h(\omega^2) = \omega^2 f(\omega^6) + \omega^4 g(\omega^{12}) = \omega^2 f(1) + \omega g(1) = 0 \quad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$$f(1)(\omega + \omega^2) + g(1)(\omega^2 + \omega) = 0$$

$\because \quad -f(1) + g(1)(-1) = 0$

$\Rightarrow \quad f(1) = -g(1) \quad \ldots(iii)$

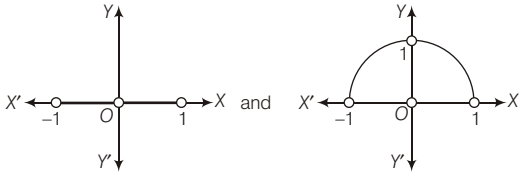
Now, on substituting $f(1)$ in Eq. (i), we get

$$-\omega g(1) + \omega^2 g(1) = 0$$

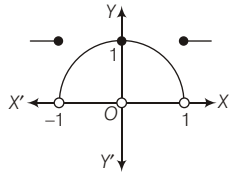
$\Rightarrow \quad g(1)(\omega^2 - \omega) = 0$

$$g(1) = 0$$

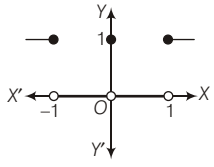
2. ***(2)*** The graph of $[|f(|x|)|], \forall\ x \in (-1, 0) \cup (0, 1)$ and $|f(x)|, \forall\ x \in (-1, 0) \cup (0, 1)$ are



$\because$ Graph of $g(x)$ is



$\therefore$ Graph of $[|g(x)|]$ is



$\Rightarrow$ Range is $\{0, 1\}$.

$\therefore$ Number of elements in the range is 2.

3. ***(1)*** We have, α, β are the roots of

$$a_1x^2 + b_1x + c_1 = a_1(x - \alpha)(x - \beta) = 0 \quad \ldots(i)$$

β, γ are the roots of

$$a_2x^2 + b_2x + c_2 = a_2(x - \beta)x - \gamma) = 0 \quad \ldots(ii)$$

and γ, α are the roots of

$$a_3x^2 + b_3x + c_3 = a_3(x - \gamma)(x - \alpha) = 0 \quad \ldots(iii)$$

Now, $\left[\prod_{r=1}^{3}\left(\frac{a_r - b_r + c_r}{a_r}\right)\right]^{\frac{1}{2}}$

$$= \left[\left(\frac{a_1 - b_1 + c_1}{a_1}\right)\left(\frac{a_2 - b_2 + c_2}{a_2}\right)\left(\frac{a_3 - b_3 + c_3}{a_3}\right)\right]^{\frac{1}{2}}$$

Now, substituting $x = -1$ in Eqs. (i), (ii) and (iii), we get

$$\left(\frac{a_1 - b_1 + c_1}{a_1}\right)\left(\frac{a_2 - b_2 + c_2}{a_2}\right)\left(\frac{a_3 - b_3 + c_3}{a_3}\right)$$

$$= (-1 - \alpha)(-1 - \beta)(-1 - \beta)(-1 - \gamma)(-1 - \gamma)(-1 - \alpha)$$

$\Rightarrow \quad \left[\prod_{r=1}^{3}\left(\frac{a_r - b_r + c_r}{a_r}\right)\right]^{\frac{1}{2}} = (1 + \alpha)(1 + \beta)(1 + \gamma)$

$$= 1 + \Sigma\alpha + \Sigma\alpha\beta + \alpha\beta\gamma$$

$\Rightarrow \quad \left[\prod_{r=1}^{3}\left(\frac{a_r - b_r + c_r}{a_r}\right)\right]^{\frac{1}{2}} - 1 = \Sigma\alpha + \Sigma\alpha\beta + \alpha\beta\gamma$

$\Rightarrow \quad k = 1$

4. **(2)** The nth term of the given sequence is

$$t_n = 1 + 2 + 2^2 + \ldots + 2^{n-1} = \frac{1(2^n - 1)}{2 - 1} = 2^n - 1$$

Now, $S_n = \Sigma t_n = \Sigma(2^n - 1) = \Sigma 2^n - \Sigma 1$

$$S_n = 2 \cdot \frac{(2^n - 1)}{2 - 1} - n = 2^{n+1} - 2 - n$$

We have, $2^{n+1} - n - 2 = 2^{n+A} + Bn^2 + Cn + D$

On comparing, we get $A = 1, B = 0, C = -1$ and $D = -2$

$\therefore \quad |A + B + C + D| = |1 + 0 - 1 - 2| = |-2| = 2$

5. **(2)** We have, $(x-1)^3 + (x-2)^3 + (x-3)^3 + \ldots + (x-2007)^3 = 0.$

There are 2007 terms and the sum gets zero. This is possible, if first and last, second and second last ….. can be cancel out.

$\Rightarrow \quad x - 1 = 2007 - x \Rightarrow x = 1004$

Hence, $\frac{k}{502} = \frac{1004}{502} = 2$

6. **(0)** We have, $|\sin x \cos x| + \sqrt{\tan^2 x + \cot^2 x + 2} = \sqrt{3}$

$\Rightarrow \quad |\sin x \cos x| + |\tan x + \cot x| = \sqrt{3}$

$\Rightarrow \quad |\sin x \cos x| + \frac{1}{|\sin x \cos x|} = \sqrt{3}$

Let $|\sin x \cdot \cos x| = t$, then $t + \frac{1}{t} = \sqrt{3}$, where $t > 0$

But $t + \frac{1}{t} = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 2 \geq 2$

Hence, $t + \frac{1}{t}$ cannot be equal to $\sqrt{3}$.

7. **(3)** We have, the geometric series $\sum_{r=0}^{\infty} \frac{a}{(y^2 - 4y + 5)^{r-1}}$

$$= a(y^2 - 4y + 5) + a + \frac{a}{(y^2 - 4y + 5)} + \frac{a}{(y^2 - 4y + 5)^2} + \ldots + \infty$$

As, we know sum of infinite geometric series is finite, if $|r| < 1$

$\therefore \quad |(y-2)^2 + 1| > 1 \Rightarrow y - 2 \neq 0 \Rightarrow y \neq 2$

$\Rightarrow \quad x^2 - 6x + 11 \neq 2 \Rightarrow x^2 - 6x + 9 \neq 0$

$\Rightarrow \quad (x-3)^2 \neq 0 \Rightarrow x \neq 3$

8. **(5)** Clearly, $\angle APB = \angle ACB = \angle C$ [$\because$ angles in the same segments of a circle are equal]

In ΔABN, $\angle 1 = \frac{\pi}{2} - \angle A$

and in ΔBAM, $\angle 2 = \frac{\pi}{2} - \angle B$

$\angle AHN = \angle 1 + \angle 2$

$= \pi - (\angle A + \angle B) = \angle C$

$\because \quad \angle AHN = \angle BHM = \angle C$

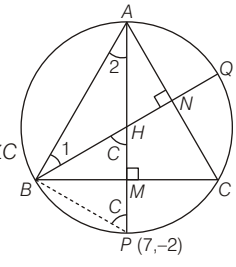
$\therefore \quad \Delta BPM \cong \Delta BHM$

Now, as $HM = MP$

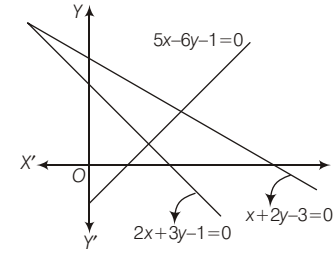
$\therefore H$ and P are mirror image in line $x = 2$.

$\Rightarrow$ Coordinates of $H = (-3, -2)$

Absolute value of sum of coordinates $= |-3-2| = 5$



9. **(a, c, d)** Origin, O and the point (α, α^2) lie on the opposite sides with respect to the line $2x + 3y - 1 = 0$



$\therefore \quad 2\alpha + 3\alpha^2 - 1 > 0$ ...(i)

Origin, O and the point (α, α^2) lie on the same side with respect to the line $x + 2y - 3 = 0$

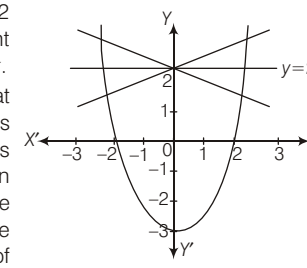
$\therefore \quad \alpha + 2\alpha^2 - 3 < 0$ ...(ii)

Again origin O and the point (α, α^2) lie on the same side with respect to the line $5x - 6y - 1 = 0$

Hence, $5\alpha - 6\alpha^2 - 1 < 0$

$\Rightarrow \quad 6\alpha^2 - 5\alpha + 1 > 0$

10. **(b, c)** The line $y = kx + 2$ passes through fixed point $(0, 2)$ for different value of k. Also, it is obvious that minimum of $A(k)$ occurs when $k = 0$, as when line is rotated from this position about point $(0, 2)$ the increased part of area more than the decreased part of area.



$\therefore$ Minimum area $= 2\int_0^{\sqrt{5}} [2 - (x^2 - 3)]\, dx$

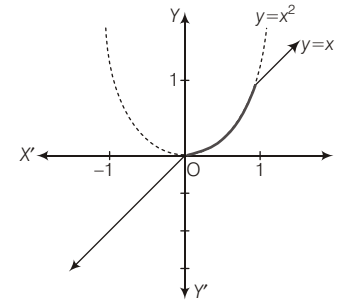
$$= 2\int_0^{\sqrt{5}} (5 - x^2)\, dx = 2\left[5x - \frac{x^3}{3}\right]_0^{\sqrt{5}}$$

$$= 2\left[5\sqrt{5} - \frac{5\sqrt{5}}{3}\right] = \frac{20\sqrt{5}}{3} \text{ sq units}$$

11. **(a, b, d)** We have,

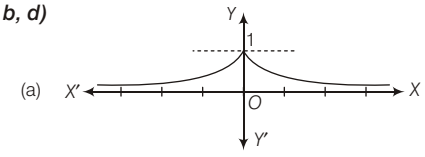
$$f(x) = \min\{x, x^2\} = \begin{cases} x, & \text{if} \quad x \leq 0 \\ x^2, & \text{if} \quad 0 < x < 1 \\ x, & \text{if} \quad x \geq 1 \end{cases}$$

The graph of above function is shown below.

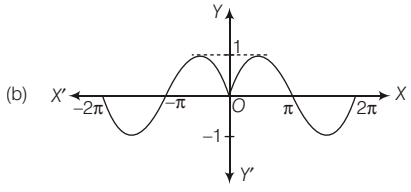


From the graph, it is clear that $f(x)$ is continuous for all $x \in R$, $f'(x) = 1$ for all $x > 1$ and f is not differentiable at $x = 0$ and $x = 1$.

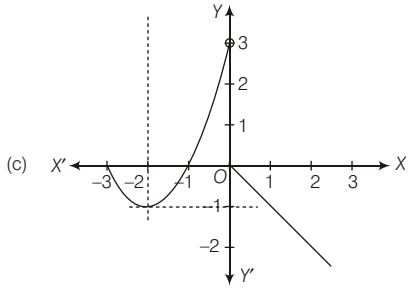
12. ***(a, b, d)***

(a) 

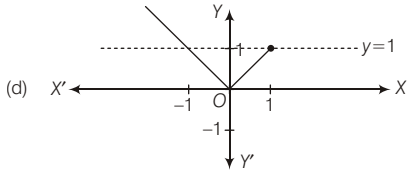
Graph of $y = e^{-|x|}$

(b) 

Graph of $y = \sin|x|$

(c) 

Graph of $f(x) = \begin{cases} x^2 + 4x + 3 & ; \quad x < 0 \\ -x & ; \quad x \geq 0 \end{cases}$

(d) 

Graph of $f(x) = \begin{cases} |x| & ; \quad x < 0 \\ \{x\} & ; \quad 0 \leq x < 1 \end{cases}$

Clearly, Fig. (a), (b) and (d) have extrema at $x = 0$.

13. ***(a, c, d)*** For continuity at $x = 1$.

Consider, $\text{RHL} = \lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} x^2 (\text{sgn}\,[x] + \{x\})$

$= 1 \cdot (1 + 0) = 1$

and $\text{LHL} = \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} x^2 (\text{sgn}\,[x] + \{x\})$

$= 1\,(\text{sgn}\,(0) + 1) = 1$

Also, $f(1) = 1$

$\therefore$ RHL = LHL = $f(1)$

Hence, $f(x)$ is continuous at $x = 1$.

Now, check differentiability at $x = 1$.

Consider, $f'(1^+) = \lim\limits_{h \to 0} \dfrac{f(1 + h) - f(1)}{h}$

$= \lim\limits_{h \to 0} \dfrac{(1 + h)^2\,(\text{sgn}[1 + h] + \{1 + h\}) - 1}{h}$

$= \lim\limits_{h \to 0} \dfrac{(1 + h)^2\,(1 + h) - 1}{h} = \lim\limits_{h \to 0} \dfrac{(1 + h)^3 - 1}{h}$

$= \lim\limits_{h \to 0} \dfrac{h^3 + 1^3 + 3h + 3h^2 - 1}{h}$

$= \lim\limits_{h \to 0} \dfrac{h^3 + 3h^2 + 3h}{h}$

$= \lim\limits_{h \to 0} h^2 + 3h + 3 = 3$

and $f'(1^-) = \lim\limits_{h \to 0} \dfrac{f(1 - h) - f(1)}{-h}$

$= \lim\limits_{h \to 0} \dfrac{(1 - h)^2\,(\text{sgn}[1 - h] + \{1 - h\}) - 1}{-h}$

$= \lim\limits_{h \to 0} \dfrac{(1 - h)^2\,(1 - h) - 1}{-h}$

$= \lim\limits_{h \to 0} \dfrac{-(h^3 + 3h - 3h^2)}{-h} = 3$

$\Rightarrow \quad f'(1^+) = f'(1^-)$

Hence, $f(x)$ is differentiable at $x = 1$.

Now, at $x = 2$,

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} x^2\,(\text{sgn}\,[x] + \{x\}) = 4\,(1 + 1) = 8$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} \sin x + |x - 3| = 1 + \sin 2$

$\Rightarrow$ LHL $\neq$ RHL

Hence, $f(x)$ is discontinuous at $x = 2$ and then $f(x)$ is also non-differentiable at $x = 2$.

14. ***(a, b)*** We have, $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = \dfrac{1}{6}\hat{\mathbf{i}} - \dfrac{1}{3}\hat{\mathbf{j}} + \dfrac{1}{3}\hat{\mathbf{k}}$

$[\mathbf{a}\ \mathbf{b}\ \mathbf{d}]\mathbf{c} - [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\mathbf{d} = \dfrac{1}{6}\hat{\mathbf{i}} - \dfrac{1}{3}\hat{\mathbf{j}} + \dfrac{1}{3}\hat{\mathbf{k}}$

$\Rightarrow \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{d}]\mathbf{c} = \dfrac{1}{6}\hat{\mathbf{i}} - \dfrac{1}{3}\hat{\mathbf{j}} + \dfrac{1}{3}\hat{\mathbf{k}} \qquad (\because [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0)$

Since, $[\mathbf{a}\ \mathbf{b}\ \mathbf{d}] = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{d}$

$= |\mathbf{a}||\mathbf{b}| \sin 30^\circ \cdot (\pm 1)$ $\quad \left[\begin{array}{l} \because \mathbf{d} \perp \mathbf{a}, \mathbf{d} \perp \mathbf{b} \\ \therefore \mathbf{d} \parallel \mathbf{a} \times \mathbf{b} \\ \Rightarrow \theta = 0 \text{ or } \pi \end{array}\right]$

$= 1 \cdot 1 \cdot \dfrac{1}{2}(\pm 1) = \pm \dfrac{1}{2}$

$\therefore \quad \mathbf{c} = \pm\left(\dfrac{1}{3}\hat{\mathbf{i}} - \dfrac{2}{3}\hat{\mathbf{j}} + \dfrac{2}{3}\hat{\mathbf{k}}\right) = \pm \dfrac{1}{3}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

15. ***(a, b, d)*** We have, $z_1 = 5 + 12i\,;\, |z_2| = 4$

$\because \quad |z_1 + iz_2| \leq |z_1| + |z_2| = 13 + 4 = 17$

$\therefore \quad \max(|z_1 + iz_2|) = 17$

$\because \quad |z_1 + (1 + i)z_2| \geq ||z_1| - |1 + i||z_2|| = 13 - 4\sqrt{2}$

$\therefore \quad \min(|z_1 + (1 + i)z_2|) = 13 - 4\sqrt{2}$

$\left|z_2 + \dfrac{4}{z_2}\right| \leq |z_2| + \dfrac{4}{|z_2|} = 4 + 1 = 5$

and $\left|z_2 + \dfrac{4}{z_2}\right| \geq |z_2| - \dfrac{4}{|z_2|} = 4 - 1 = 3$

$\therefore \max\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{3}$ and $\min\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{5}$

16. ***(a, c, d)*** The required probability

$$P(E) = \frac{^{2n}C_n}{2^{2n}} = \frac{(2n)!}{n!\,n!2^n \cdot 2^n} = \frac{1 \times 2 \times 3 \times \ldots \times 2n}{n!\,n!\,2^n\,2^n}$$

$$= \frac{(1 \times 3 \times 5 \times \ldots \times (2n-1))\,2^n\,n!}{n!n!2^n\,2^n}$$

$$= \frac{(1 \times 3 \times 5 \times \ldots \times (2n-1))}{n!\,2^n}$$

Note that, $\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right) = \frac{1 \times 3 \times 5 \times \ldots \times (2n-1)}{2 \times 4 \times 6 \times \ldots \times (2n)}$

$$\sum_{r=0}^{n}\left(\frac{^nC_r}{2^n}\right)^2 = \frac{1}{2^n} \cdot \frac{1}{2^n} \sum_{r=0}^{n}(^nC_r)^2 = \frac{^{2n}C_n}{2^{2n}}$$

and $$\frac{\sum_{r=0}^{n}(^nC_r)^2}{\left(\sum_{r=0}^{2n} {}^{2n}C_r\right)} = \frac{^{2n}C_n}{2^{2n}}$$

17. ***(a, b, d)*** We have hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

whose foci are $(\pm\, ae, 0)$

and ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ whose foci are $(\pm\, 4, 0)$.

Given, $ae = 4$ and $e = 2$

$\therefore\ b^2 = 4\,(4-1) = 12 \Rightarrow b = \sqrt{12}$ $\quad [\because a = 2]$

Equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Its latusrectum $= \frac{2b^2}{a} = \frac{2 \times 12}{2} = 12$

Also, note that there is no director circle, as equation of director circle is given by $x^2 + y^2 = a^2 - b^2$.

18. ***(a, d)*** We have, $\int \frac{e^{x-1}}{(x^2 - 5x + 4)}\,2x\,dx = I$ (say)

Take $$\frac{2x}{x^2 - 5x + 4} = \frac{P}{x-1} + \frac{Q}{x-4}$$

$\Rightarrow$ $$P = -\frac{2}{3} \text{ and } Q = \frac{8}{3}$$

$\therefore$ $$I = \int e^{x-1}\left[-\frac{2}{3(x-1)} + \frac{8}{3(x-4)}\right]dx$$

$$= -\frac{2}{3}f(x-1) + \frac{8}{3}f(x-4) + C$$

$\Rightarrow$ $$A = -\frac{2}{3} \text{ and } B = \frac{8}{3}e^3$$

19. ***(a)*** **A $\rightarrow$ s, t; B $\rightarrow$ p, t; C $\rightarrow$ q; D $\rightarrow$ r**

A. Both lines pass through the point (7, 11, 15).

B. $<2, 3, 4>$ are direction ratios of both the lines. Also, the point (1, 2, 3) is common to both.

$\therefore$ The lines are coincident.

C. $<5, 4, -2>$ are direction ratios of both the lines.

$\therefore$ The lines are parallel.

Also, $x = 2 + 5\lambda$, $y = -3 + 4\lambda$, $z = 5 - 2\lambda$

$$\therefore \frac{2 + 5\lambda - 7}{5} = \frac{-3 + 4\lambda - 1}{4} = \frac{5 - 2\lambda - 2}{-2}$$

i.e. $\lambda - 1 = \lambda - 1 = \frac{3 - 2\lambda}{-2}$

i.e. no value of λ.

Thus, the lines are different and parallel.

D. $<2, 3, 5>$ and $<3, 2, 5>$ are direction ratios of first and second line, respectively.

$\therefore$ The lines are not parallel.

Now, $x = 3 + 2\lambda$; $y = -2 + 3\lambda$; $z = 4 + 5\lambda$

$x = 3 + 3\mu$; $y = -2 + 2\mu$; $z = 7 + 5\mu$

are parametric equations of the lines.

On solving

$3 + 2\lambda = 3 + 3\mu$

and $-2 + 3\lambda = 2 + 2\mu$, we get

$$\lambda = \frac{12}{5}, \mu = \frac{8}{5}$$

Now, substituting these values in

$4 + 5\lambda = 7 + 5\mu$, we get $4 + 12 = 7 + 8, 16 = 15$

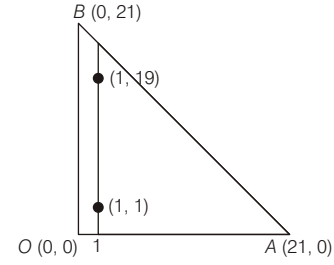
which is not true.

$\therefore$ The lines do not intersect.

Hence, the lines are skew.

20. **A $\rightarrow$ p,q, r; B $\rightarrow$ p,q,r; C $\rightarrow$ p,q,r,s,t,; D $\rightarrow$ p**

A.



Let the vertices of triangle be O (0, 0), A (21, 0) and B (0, 21). Now, the integral points lying on the line $x = 1$ and satisfy $x + y < 21$ are (1, 1), (1, 2), ..., (1, 19). Thus, there are 19 points lying on the line $x = 1$.

Similarly, the number of integral points on the line $x = 2$ is 18 and on $x = 3$ is 17 and so on. The number of integral points on $x = 19$ is just 1. Hence, the number of integral points lying in the interior of the triangle

$$= 19 + 18 + 17 + \ldots + 1 = \frac{19 \cdot (19+1)}{2} = 190$$

B. Numbers of triangles formed by n points in which p points are collinear $= {}^nC_3 - {}^pC_3$.

$\therefore$ Required number of triangles $= {}^{12}C_3 - {}^7C_3 = 185$

C. Required number of subsets = The total number of subsets − The number of subsets having only even numbers $= 2^9 - 2^4 = 496$

D. Factorising the given number, we have

$$38808 = 2^3 \cdot 3^2 \cdot 7^2 \cdot 11$$

The total number of divisors of this number is same as the number of ways of selecting some or all of three 2's, two 3's, two 7's and one 11.

$\therefore$ The total number of divisors

$$= (3+1)(2+1)(2+1)(1+1) = 72$$

Hence, the required number of divisors $= 72 - 2 = 70$

Paper II

1. **(7)** $\frac{\log x}{2} = \frac{3}{\log y} = \frac{\log z}{5} = \frac{\log x + 3 + \log z}{2 + \log y + 5}$

$= \frac{\log zx + 3}{7 + \log y} = \frac{1 + 3}{7 + \log y} = \frac{4}{\log y + 7}$

$\Rightarrow \frac{3}{\log y} = \frac{4}{7 + \log y} \Rightarrow 21 + 3\log y = 4\log y$

$\Rightarrow \log y = 21 \Rightarrow y = 10^{21}$

$\therefore \log\left(\frac{y}{(zx)^{14}}\right) = \log\left(\frac{10^{21}}{10^{14}}\right) = \log 10^7 = 7$

2. **(2)** In a ΔABC, $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

$\Rightarrow \frac{3}{2} + 3k = \frac{1}{2}\left(\frac{2k+1}{2}\right)\left(\frac{4k+1}{2}\right)$

$\Rightarrow \frac{3 + 6k}{2} = \frac{8k^2 + 6k + 1}{8}$

$\Rightarrow 12 + 24k = 8k^2 + 6k + 1$

$\Rightarrow 8k^2 - 18k - 11 = 0$

$\Rightarrow 8k^2 - 22k + 4k - 11 = 0$

$\Rightarrow (2k+1)(4k-11) = 0 \Rightarrow k = -\frac{1}{2}$ or $\frac{11}{4}$

For $k = -\frac{1}{2}$, $\tan B = 0$ not possible

$\therefore k = \frac{11}{4} \Rightarrow [k] = 2$

3. **(9)** Since, $\mathbf{x}$ and $\mathbf{y}$ are non-collinear vectors, therefore $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{x} \times \mathbf{y}$ are non-coplanar vectors.

Now, consider,

$[(a-2)\alpha^2 + (b-3)\alpha + c]\mathbf{x} + [(a-2)\beta^2 + (b-3)\beta + c]\mathbf{y}$

$+ [(a-2)\gamma^2 + (b-3)\gamma + c](\mathbf{x} \times \mathbf{y}) = 0$

$\Rightarrow$ Coefficient of each vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{x} \times \mathbf{y}$ is zero.

[$\because \mathbf{x}$, $\mathbf{y}$ and $\mathbf{x} \times \mathbf{y}$ are non-coplanar, so linearly independent]

$(a-2)\alpha^2 + (b-3)\alpha + c = 0$

$(a-2)\beta^2 + (b-3)\beta + c = 0$

$(a-2)\gamma^2 + (b-3)\gamma + c = 0$

The above three equations will satisfy, if the coefficients of α, β and γ are zero because α, β and γ are three distinct real numbers.

So, we have $a - 2 = 0 \Rightarrow a = 2$

$b - 3 = 0$

$\Rightarrow b = 3$ and $c = 0$

$\therefore a^2 + b^2 + c^2 = 4 + 9 + 0 = 13$

$\Rightarrow a^2 + b^2 + c^2 - 4 = 9$

4. **(3)** Let the given points be $A(1,0,-1)$ and $B(-1,2,2)$.

Now, direction ratios of AB are $< -2,2,3 >$. Let θ be the angle between the line and normal to the plane, then

$$\sin\theta = \left|\frac{(-2)1 + 2(3) + 3(-5)}{\sqrt{1+9+25}\sqrt{4+4+9}}\right| = \frac{11}{\sqrt{17}\sqrt{35}} = \frac{11}{\sqrt{595}}$$

Length of projection $= (AB)\cos\theta$

$\Rightarrow d = \sqrt{(-2)^2 + (2)^2 + (3)^2} \times \sqrt{1 - \frac{121}{595}} = \sqrt{17}\sqrt{\frac{474}{595}}$

$\therefore \frac{d}{2} = \sqrt{\frac{474 \times 17}{595 \times 4}} \Rightarrow \left[\frac{d}{2}\right] = 3$

5. **(5)** We have,

$$(\cot^{-1}x)(\tan^{-1}x) + \left(2 - \frac{\pi}{2}\right)\cot^{-1}x - 3\tan^{-1}x - 3\left(2 - \frac{\pi}{2}\right) > 0$$

Now put $\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$

$\Rightarrow \cot^{-1}x\left(\frac{\pi}{2} - \cot^{-1}x\right) + \left(2 - \frac{\pi}{2}\right)\cot^{-1}x - 3\left(\frac{\pi}{2} - \cot^{-1}x\right) - 6 + \frac{3\pi}{2} > 0$

$\Rightarrow -(\cot^{-1}x)^2 + 2\cot^{-1}x + 3\cot^{-1}x - 6 > 0$

$\Rightarrow (\cot^{-1}x - 3)(2 - \cot^{-1}x) > 0$

$\Rightarrow (\cot^{-1}x - 3)(\cot^{-1}x - 2) < 0$

$\Rightarrow 2 < \cot^{-1}x < 3$

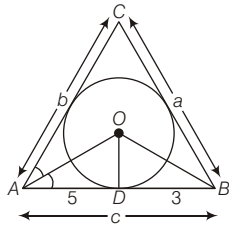
$\Rightarrow \cot 3 < x < \cot 2$

Hence, $x \in (\cot 3, \cot 2)$

$\Rightarrow \cot^{-1}a + \cot^{-1}b = \cot^{-1}(\cot 3) + \cot^{-1}(\cot 2)$

$= 3 + 2 = 5$

6. **(4)**



We have, $\angle A = 60°$

$\Rightarrow \tan 30° = \frac{r}{5} \Rightarrow r = \frac{5}{\sqrt{3}}$

Also, we have $\tan\frac{B}{2} = \frac{r}{3} = \frac{5}{3\sqrt{3}}$

Now, $\cos B = \frac{1 - \tan^2\frac{B}{2}}{1 + \tan^2\frac{B}{2}} = \frac{1 - \frac{25}{27}}{1 + \frac{25}{27}} = \frac{2}{52} = \frac{1}{26}$

$\Rightarrow \sin B = \frac{15\sqrt{3}}{26}$

Also, $\sin C = \sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$= \frac{\sqrt{3}}{2} \times \frac{1}{26} + \frac{1}{2} \times \frac{15\sqrt{3}}{26}$

$= \frac{1}{52} \times 16\sqrt{3} = \frac{4\sqrt{3}}{13}$

Now, $\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow a = \frac{8\sqrt{3}}{2} \times \frac{13}{4\sqrt{3}} = 13$

$\therefore \left[\frac{BC}{3}\right] = \left[\frac{13}{3}\right] = 4$

7. ***(4)*** $f(x) = ax^2 - (3 + 2a)x + 6 = (ax - 3)(x - 2)$

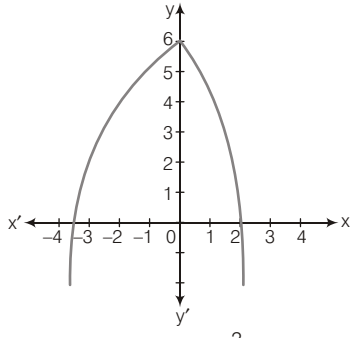
Here, roots of the equation $f(x) = 0$ are 2 and $\frac{3}{a}$ and $f(0) = 6$.

$f(x)$ should be positive for exactly three negative integral values of x.

Therefore, graph of $f(x)$ must be a downward parabola passing through $x = 2$ and $x = \frac{3}{a}$,

and $$-4 \le \frac{3}{a} < -3$$

$\therefore$ $$a \in \left(-1, -\frac{3}{4}\right]$$



$\therefore$ $$c = -1, d = -\frac{3}{4}$$

$\Rightarrow$ $$c^2 + 4|d| = 1 + 3 = 4$$

8. ***(4)*** Let $\Delta = \begin{vmatrix} x^n & x^{n+2} & x^{n+4} \\ y^n & y^{n+2} & y^{n+4} \\ z^n & z^{n+2} & z^{n+4} \end{vmatrix} \Rightarrow \Delta = x^n y^n z^n \begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix}$

$$= (xyz)^n (x^2 - y^2)(y^2 - z^2)(z^2 - x^2)$$

Clearly, when $n = -4$, then

$$\Delta = \left(\frac{1}{y^2} - \frac{1}{x^2}\right)\left(\frac{1}{z^2} - \frac{1}{y^2}\right)\left(\frac{1}{x^2} - \frac{1}{z^2}\right)$$

9. ***(a, b, d)*** We have,

$$f(m) = \sum_{i=0}^{m} ({}^{30}C_{30-i})({}^{20}C_{m-i}) = \sum_{i=0}^{m} {}^{30}C_i \, {}^{20}C_{m-i}$$

$$= {}^{30}C_0 \cdot {}^{20}C_m + {}^{30}C_1 \cdot {}^{20}C_{m-1} + \ldots + {}^{30}C_m \cdot {}^{20}C_0$$

It is the coefficient of x^m in $(1 + x)^{30} \cdot (1 + x)^{20} = (1 + x)^{50}$

$\therefore$ $f(m) = {}^{50}C_m$

Clearly, $f(m)$ is the greatest when $m = 25$, also

$$f(0) + f(1) + \ldots + f(50)$$
$$= {}^{50}C_0 + {}^{50}C_1 + {}^{50}C_2 + \ldots + {}^{50}C_{50} = 2^{50}$$

Also, ${}^{50}C_m$ is not divisible by 50 for any m as 50 is not a prime number

Now, $\sum_{m=0}^{50} [f(m)]^2 = ({}^{50}C_0)^2 + ({}^{50}C_1)^2 + \ldots + ({}^{50}C_{50})^2 = {}^{100}C_{50}$

10. ***(a, b)*** We have,

$$L_1 L_2 + \lambda L_2 L_3 + \mu L_3 L_1 = 0$$

$\Rightarrow$ $$(y - m_1 x)(y - m_2 x) + \lambda (y - m_2 x)(y - m_3 x) + \mu (y - m_3 x)(y - m_1 x) = 0 \quad \ldots(i)$$

Clearly, Eq. (i) represents a curve passing through points intersection of lines L_1, L_2 and L_3. Eq. (i) will represent a circle, if coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$.

$\therefore$ $$1 + \lambda + \mu = m_1 m_2 + \lambda m_2 m_3 + \mu m_1 m_3$$

and $$m_1(1 + \mu) + m_2(1 + \lambda) + m_3(\lambda + \mu) = 0$$

11. ***(a, b, c)*** We have,

$$f(x) = \cos^{-1} x + \cos^{-1}\left[\frac{\sqrt{3}}{2}x + \sqrt{1 - \frac{3}{4}}\sqrt{1 - x^2}\right]$$

$$\Rightarrow f(x) = \begin{cases} 2\cos^{-1} x - \frac{\pi}{6} & ; \quad -1 \le x \le \frac{\sqrt{3}}{2} \\ \cos^{-1} x + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - \cos^{-1} x & ; \quad \frac{\sqrt{3}}{2} \le x \le 1 \end{cases}$$

Now, $$\sum_{r=1}^{100} f\left[1 - \left(\frac{1}{10}\right)^r\right] = f\left(\frac{9}{10}\right) + f\left(\frac{99}{10^2}\right) + f\left(\frac{999}{10^3}\right) + \ldots + f\left(\frac{999\ldots9}{10^{100}}\right)$$

$$= \frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{6} + \ldots + \frac{\pi}{6} = 100 \cdot \frac{\pi}{6} = \frac{50}{3}\pi$$

$\Rightarrow$ $$p - 16q = 50 - 16 \times 3 = 50 - 48 = 2$$

12. ***(b, c)*** The number of ways of inviting, with the couple not included, is ${}^8C_5$.

The number of ways of inviting, with the couple included, is ${}^8C_3$.

Therefore, the required number of ways is

$${}^8C_5 + {}^8C_3 = {}^8C_3 + {}^8C_3 = 2 \times {}^8C_3$$

Also, $${}^{10}C_5 - 2 \times {}^8C_4 = \frac{10!}{5!5!} - 2 \times \frac{8!}{4!4!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{120} - \frac{2 \times 8 \times 7 \times 6 \times 5}{24}$$

$$= 9 \times 4 \times 7 - 140$$

$$= 112 = 2 \times \frac{8!}{3!\,5!}$$

13. ***(c, d)*** Parametric relation, if normal at any point on the parabola meets the parabola again, is

$$t_2 = -t_1 - \frac{2}{t_1}$$

Also, $$\frac{2at_1}{at_1^2} \times \frac{2at_2}{at_2^2} = -1$$

$\Rightarrow$ $$t_1 t_2 = -4$$

$\Rightarrow$ $$-\frac{4}{t_1} = -t_1 - \frac{2}{t_1}$$

$\Rightarrow$ $$t_1^2 + 2 = 4$$

and $$t_1 = \pm\sqrt{2}$$

So, points can be $(2a, \pm 2\sqrt{2}a)$.

14. ***(a, c, d)*** $f^2(x) = f\left(\frac{3x}{4} + 1\right) = \frac{3}{4}\left(\frac{3x}{4} + 1\right) + 1$

$$= \left(\frac{3}{4}\right)^2 x + \frac{3}{4} + 1$$

$$f^3(x) = f\{f^2(x)\} = \frac{3}{4}\{f^2(x) + 1\}$$

$$= \frac{3}{4}\left\{\left(\frac{3}{4}\right)^2 x + \frac{3}{4} + 1\right\} + 1$$

$$= \left(\frac{3}{4}\right)^3 x + \left(\frac{3}{4}\right)^2 + \frac{3}{4} + 1$$

$$\therefore\ f^n(x) = \left(\frac{3}{4}\right)^n x + \left(\frac{3}{4}\right)^{n-1} + \left(\frac{3}{4}\right)^{n-2} + \ldots + \left(\frac{3}{4}\right) + 1$$

$$= \left(\frac{3}{4}\right)^n x + \frac{1 - \left(\frac{3}{4}\right)^n}{1 - \frac{3}{4}}$$

$\therefore\ \lambda = \lim\limits_{n \to \infty} f^n(x) = 0 + 4 = 4$

15. ***(a, c)*** $\frac{d}{dx}\{f_n(x)\} = \frac{d}{dx}\{e^{f_{n-1}(x)}\}$

$= e^{f_{n-1}(x)} \frac{d}{dx}\{f_{n-1}(x)\} = f_n(x) \frac{d}{dx}\{f_{n-1}(x)\}$

$= f_n(x) \frac{d}{dx}\{e^{f_{n-2}(x)}\} = f_n(x) \cdot e^{f_{n-2}(x)} \frac{d}{dx}\{f_{n-2}(x)\}$

$= f_n(x) f_{n-1}(x) \frac{d}{dx}\{f_{n-2}(x)\}$

.....

.....

$= f_n(x) f_{n-1}(x) \ldots f_2(x) \frac{d}{dx}\{f_1(x)\}$

$= f_n(x) \cdot f_{n-1}(x) \ldots f_2(x) \frac{d}{dx}\{e^{f_0(x)}\}$

$= f_n(x) \cdot f_{n-1}(x) \ldots f_2(x) e^{f_0(x)} \frac{d}{dx}\{f_0(x)\}$

Using $e^{f_0(x)} = f_1(x)$ and $f_0(x) = x$, we get

$\frac{d}{dx}\{f_n(x)\} = f_n(x) f_{n-1}(x) \ldots f_2(x) \cdot f_1(x)$

16. ***(a, c)*** Two circle intersect orthogonally, if $2gg' + 2ff' = c + c'$

$\Rightarrow \quad 2 \times 1 \times (0) + 2k \cdot k = 6 + k$

$\Rightarrow \quad 2k^2 - k - 6 = 0$

$\Rightarrow \quad 2k^2 - 4k + 3k - 6 = 0$

$\Rightarrow \quad 2k(k-2) + 3(k-2) = 0$

$\Rightarrow \quad (k-2)(2k+3) = 0$

$\therefore \quad k = 2, -\frac{3}{2}$

17. ***(a)*** The total number of ways of painting first column when colours are not alternating is $2^8 - 2$.

The total number of ways when no column has alternating colours is $\frac{(2^8 - 2)^8}{2^{64}}$.

18. ***(a)*** Number of ways, the square has equal number of red and black squares, is ${}^{64}C_{32}$.

Hence, the required probability is $\frac{{}^{64}C_{32}}{2^{64}}$.

19. ***(d)*** Let $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

Then, $\mathbf{r} \cdot (10\hat{\mathbf{j}} - 8\hat{\mathbf{i}} - \mathbf{r}) = 40$ becomes

$x^2 + y^2 + 8x - 10y + 40 = 0$ which is a circle with

centre $(-4, 5)$; radius $= 1$

$p_1 = \max\{(x+2)^2 + (y-3)^2\}$

$p_2 = \min\{(x+2)^2 + (y-3)^2\}$

Let P be $(-2, 3)$. Then,

$CP = 2\sqrt{2}, r = 1$

$p_2 = (2\sqrt{2} - 1)^2$

$p_1 = (2\sqrt{2} + 1)^2$

20. ***(c)*** Slope of $AB = \left(\frac{dy}{dx}\right)_{(2,2)} = -2$

Equation of AB, $2x + y = 6$

$\mathbf{OA} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$, $\mathbf{OB} = 3\hat{\mathbf{i}}$

$\mathbf{AB} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$\mathbf{AB} \cdot \mathbf{OB} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) \cdot (3\hat{\mathbf{i}}) = 3$

Notice Board

No weightage to class XII marks from 2017

HRD minister officials said that, class XII marks will not be a factor for determining rankings in the Joint Entrance Examination (JEE) for admissions to engineering undergraduate courses from next year.

Class XII scores will be a crucial parameter only for determining the eligibility criteria for the exam. To qualify for appearing in exam, students need to have atleast 75% marks or be in top 20 percentile in class XII examination. SC, ST students should have 65% marks.

Also, there will not be any change in the existing two-tier pattern of JEE Main and Advanced.

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Mock Questions with Complete Solutions

Paper 1

Section I

Single Digit Integer Type

1. Let $p(x) = x^5 + x^2 + 1$ have roots x_1, x_2, x_3, x_4 and x_5, $g(x) = x^2 - 2$, then find the value of $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5) - 30g(x_1x_2x_3x_4x_5)$

2. If $\tan\frac{\pi}{8}$ is the root of the equation $x^4 - kx^2 + 1 = 0$, then the value of k is

3. If the differential equation of curve, passing through $\left(0, \frac{-\pi}{4}\right)$ and $(t, 0)$ is $\cos y\left(\frac{dy}{dx} + e^{-x}\right) + \sin y\left(e^{-x} - \frac{dy}{dx}\right) = e^{e^{-x}}$, then the value of $t \cdot e^{e^{-t}}$ is

4. Two lines $zi - \bar{z}i + 2 = 0$ and $z(1+i) + \bar{z}(1-i) + 2 = 0$ intersect at a point P. There is a complex number $\alpha = x + iy$ at a distance of 2 units from the point P lies on line $z(1+i) + \bar{z}(1-i) + 2 = 0$, then the value of $[|x|]$ is (where, $[\cdot]$ represents greatest integer function)

5. Suppose that there are 5 red points and 4 blue points on a circle. Let $\frac{m}{n}$ be the probability that a convex polygon whose vertices are among the 9 points has atleast one blue vertex, where m and n are relatively prime, then the value of $|n - m|$ is

6. A is a 2×2 matrix such that $A\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}-1\\2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$. If the sum of the elements of matrix A is k, then the value of k is

7. If $1\cdot0! + 3\cdot1! + 7\cdot2! + 13\cdot3! + \ldots$ to $(n+1)$ terms $= 4000\,(4000)!$, then the value of $\frac{n+1}{500}$ is

8. Let O be an interior point of ΔABC such that $\mathbf{OA} + 2\mathbf{OB} + 3\mathbf{OC} = \mathbf{0}$. If the ratio of area of ΔABC to area of ΔAOC is $m : n$. Then, the value of $m + n$ is

Section II

One or More than One Options Correct Type

9. Let a, b, c be three real numbers such that $1 \geq a \geq b \geq c \geq 0$. If λ (real or complex) is a root of the cubic equations $x^3 + ax^2 + bx + c = 0$, then
(a) $\lambda = 1$ *(b)* $|\lambda| < 1$ *(c)* $|\lambda| > 1$ *(d)* $\lambda = -1$

10. If $f(x) = [x^{-2}[x^2]]$, (where $[\cdot]$ denotes the greatest integer function) $x \neq 0$, then
(a) $f(x)$ is continuous everywhere
(b) $f(x)$ is discontinuous at $x = \sqrt{2}$
(c) $f(x)$ is non-differentiable at $x = 1$
(d) $f(x)$ is discontinuous at infinitly many points

11. The vector $\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is rotated through an angle θ and doubled in magnitude. It now becomes $4\hat{\mathbf{i}} + (4x-2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The values of x are
(a) 1 *(b)* $-\frac{2}{3}$ *(c)* 2 *(d)* $\frac{4}{3}$

12. Let a function $f : R \to R$ is defined by $f(x) = x + \sin x$ and $\int_0^{\pi} f^{-1}(x)dx = I$, then

(a) $I > \int_0^1 \frac{1}{1+x^3}dx$

(b) $I < \int_0^1 e^{x^2}dx$

(c) $2 < I < 3$

(d) $\frac{\pi}{4} < I < \frac{\pi}{2}$

13. If in a triangle $\sin^2 B \sin^2 C + 2\sin^2 C \sin^2 A + 2\sin^2 A \sin^2 B = \sin^4 A + \sin^4 B + \sin^4 C$, then its angle A is equal to

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$

(c) $\frac{5\pi}{6}$ (d) $\frac{7\pi}{6}$

14. The number of words formed with or without meaning with each of 2 vowels and 3 consonants taken from the letters of the word 'EVOLUTION' is written in the form $2^a \cdot 3^b \cdot 5^c \cdot 13^d$, then

(a) $a + b = 5$

(b) $a + c = 5$

(c) $c + d = 2$

(d) $a + b + c = 6$

15. The equation of the plane whose distance from the origin is unity and which is passes through the line $x + 3y + 6 = 0 = 3x - y - 4z$ is

(a) $x + 2y + 2z - 3 = 0$

(b) $2x + y + 2z - 3 = 0$

(c) $x - 2y - 2z - 3 = 0$

(d) $2x + y - 2z + 3 = 0$

16. A number of the form xyzxyz is divisible by

(a) 7 (b) 11

(c) 13 (d) None of these

17. The value of $\int_{\lambda}^{\lambda + \pi/2} (\cos^4 x + \sin^4 x)dx$ is

(a) $\lambda\left(\frac{\pi}{2}\right)^2$ (b) independent of λ

(c) $\frac{3\pi}{8}$ (d) $\left(\frac{3\pi}{8}\right)\lambda^2$

18. Let A, B, C be $n \times n$ real matrices and product is pairwise commutative, also $ABC = O_n$. If $\lambda = \det(A^3 + B^3 + C^3) \cdot \det(A + B + C)$, then

(a) $\lambda > 0$ (b) $\lambda < 0$

(c) $\lambda = 0$ (d) $\lambda \in (-\infty, \infty) - \{0\}$

Section III

Match the Columns Type

19. Match the statements of Column I with the values of Column II.

	Column I	Column II
A.	If $0 < a, b < 8$ and $\frac{x^2+5}{2} = x - 2\cos(ax + b)$ has atleast one real solution, then the greatest value of $a + b$ is	p. 2π
B.	The sum of the infinite series $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{4}{33} + \ldots$ is	q. $\frac{3\pi}{2}$
C.	If $\sin^{-1}\sqrt{x^2+2x+1} + \sec^{-1}\sqrt{x^2+2x+1} = \frac{\pi}{2}$, then the value of $2\sec^{-1}\left(\frac{x}{2}\right) + \sin^{-1}\left(\frac{x}{2}\right)$ is	r. 5π
D.	If $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$, then the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is	s. $\pi/4$

20. Match the statements of Column I with the values of Column II.

	Column I	Column II
A.	Using all 3 distinct real numbers matrices each having distinct elements and of all possible orders are to be made, then the number of possible matrices are	p. 6
B.	Given six line segments of lengths 2, 3, 4, 5, 6, 7 units, the number of triangles that can be formed by these line segments is	q. 8
C.	Let $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ be unit vectors such that $\mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{a}$, $\mathbf{x} \times (\mathbf{y} \times \mathbf{z}) = \mathbf{b}$, $(\mathbf{x} \times \mathbf{y}) \times \mathbf{z} = \mathbf{c}$, $\mathbf{a} \cdot \mathbf{x} = \frac{3}{2}$, $\mathbf{a} \cdot \mathbf{y} = \frac{7}{4}$ and $\lvert\mathbf{a}\rvert = 2$. If $\mathbf{y} = -\lambda\mathbf{c}$, then 2λ must be	r. 13
D.	The sum of the series $\frac{1}{\sin 45^\circ \sin 46^\circ} + \frac{1}{\sin 46^\circ \sin 47^\circ} + \frac{1}{\sin 47^\circ \sin 48^\circ} + \ldots \frac{1}{\sin 134^\circ \sin 135^\circ}$ is $n \cdot \operatorname{cosec} 1^\circ$, then $3n$ is equal to	s. 12

Think About it!

If f is differentiable everywhere and has the three properties

(i) $f(x_1 + x_2) = f(x_1)f(x_2)$ (ii) $f(0) = 1$ (iii) $f'(0) = 1$, then $f'(x) = f(x)$ for all x.

"It is true".

Paper 2

Section I

Single Digit Integer Type

1. If the expression $z^5 - 32$ can be factorized into linear and quadratic factors over real coefficients as $(z^5 - 32) = (z-2)(z^2 - pz + 4)\ (z^2 - qz + 4)$, then the value of $(p^2 + 2p)$ is

2. Let P_i and P_i' be the feet of the perpendicular drawn from foci S, S' on a tangent T_i to an ellipse whose length of semi-major axis is 20.
If $\sum_{i=1}^{10} (SP_i)(S'P_i') = 2560$, then the value of $5e$ (where, e is eccentricity of ellipse) is

3. Let $P(x) = x^2 + bx + c$, where b and c are integers. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then the value of $P(1)$ is

4. Let $f(x) = \int_0^x \sin(t^2 - t + x)\,dt$, then the value of $f''\left(\frac{-1}{2}\right) + f\left(\frac{-1}{2}\right)$ is

5. If ${}^9C_n + 3\cdot{}^9C_{n+1} + 3\cdot{}^9C_{n+2} + {}^9C_{n+3} > {}^{12}C_{n+2}$, then the greatest value of n is

6. Suppose a matrix A satisfies $A^2 - 5A + 7I = O$, if $A^6 = aA + bI$, then the value of $\left[\frac{a-b}{150}\right]$ (where, $[\cdot]$ denotes greatest integer function) is

7. From a point P outside a circle with centre at C, tangents PA and PB are drawn such that $\frac{1}{(CA)^2} + \frac{1}{(PA)^2} = \frac{1}{16}$, then the length of chord AB is

8. If a and b are positive numbers and $\lim_{x\to 0} \frac{1 + a^3 + 8e^{1/x}}{1 + (2 + b + b^2)e^{1/x}} = 2$, then the value of $a^2 + b^2$ is

Section II

One or More than One Options Correct Type

9. A tangent is drawn at any point (l, m), $l, m \neq 0$ on the parabola $y^2 = 4ax$ and the tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ (where, $a > 0$) such that all chords of contact pass through a fixed point (α, β), then
(a) $\alpha l > 0, \beta m > 0$ *(b)* $\alpha l < 0, \beta m > 0$
(c) $2\left(\frac{l}{\alpha}\right)^2 + \left(\frac{m}{\beta}\right)^2 = 8$ *(d)* $4\left(\frac{l}{\alpha}\right) + \left(\frac{m}{\beta}\right)^2 = 0$

10. If a_i is negative or positive according as i is odd or even and equation $x^4 + a_1x^3 + a_2x^2 + a_3x + 4 = 0$ has four real roots, then
(a) minimum value of a_1a_3 is 16
(b) minimum value of a_2 is 12
(c) minimum value of a_1a_3 is 64
(d) minimum value of a_2 is 16

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then
(a) $A^3 - A^2 = A - I$
(b) $\det(A^{2016} - I) = 0$
(c) $A^{2016} = \begin{bmatrix} 1 & 0 & 0 \\ 1008 & 1 & 0 \\ 1008 & 0 & 1 \end{bmatrix}$
(d) $A^{2016} = \begin{bmatrix} 1 & 1 & 0 \\ 1008 & 1 & 0 \\ 0 & 0 & 1008 \end{bmatrix}$

12. Given functional equation $f(x) + \frac{3}{2} f\left(\frac{2x+29}{x-2}\right) = 50x + 40$. Now if $f(x) = A - Bx + \frac{5C}{x-2}$, $x \neq 2$, then
(a) $\left(2000 - \frac{B}{2}\right) = 5C$
(b) $17A = 3B$
(c) $99A = 34C$
(d) $5A = 17B$

13. Consider the equation
$$\sqrt{1 + \frac{3\cos 2x}{\sqrt{\sin^4 x + 4\cos^2 x} + \sqrt{\cos^4 x + 4\sin^2 x}}} = \sqrt{2}\sin^{-1}(\sin 2x),\ x \in [-\pi, \pi]$$
(a) Number of roots of this equation is 2
(b) Number of roots of this equation is 4
(c) One root lies in the interval $\left(\frac{\pi}{8}, \frac{\pi}{6}\right)$
(d) If $x \in [0, n\pi]$, $n \in N$, then number of roots will never be odd

14. Given, $f(x) - \mu\int_0^{\pi/4} \sin^2 x \cos^{-5} x \cos t\, f(t)\,dt = \sin^2 x \cos^{-5} x$, where μ is a constant, then
(a) $f(x) = \frac{3}{3-\mu}\sin^2 x \cos^{-5} x$
(b) $f(x) = \frac{3}{\mu}\sin x \cos x$
(c) $f(x)$ is periodic with period π
(d) $f(x)$ is periodic with period 2π

15. $\int_a^b \text{sgn}(x)dx$ is equal to

(a) $|b| - |a|$
(b) $(b - a)\text{sgn}(b - a)$
(c) $b\,\text{sgn}(b) - a\,\text{sgn}(a)$
(d) $|a| - |b|$

16. Let k_1, k_2 be two integers such that $(n - a)! = (n - b)!$, $2a + b + 1 = k_1 n + k_2$, $\forall\, n$, where $a < b \le n$ and $a, b, n \in N$. Let P and Q be two points on the curve $y = \log_{1/2}\left(x + \frac{k_2}{2}\right) + \log_2 \sqrt{4x^2 + 4k_2 x + k_1 + k_2}$.

Point P also lies on the circle $x^2 + y^2 = k_1^3 - 2k_2$. However Q lies inside the circle such that its abscissa is an integer, then

(a) the value of k_1 and k_2 are respectively 2 and –1
(b) maximum value of $\mathbf{OP} \cdot \mathbf{OQ}$ is 7
(c) minimum value of $|\mathbf{PQ}|$ is 1
(d) minimum value of $\mathbf{OP} \cdot \mathbf{OQ}$ is 3

Section III

Linked Comprehension Type

Passage I

Directions (Q. Nos. 17-18) Let $f : R \to R$ and $f(1) = 1$, $f(2) = 4$, $f(3) = 9$

17. If $f''(x)$ exists, $\forall\, x \in R$, then

(a) $f''(x) = 2$ for $\forall\, x \in [1, 3]$
(b) $f''(x) = 2$ for some $x \in [1, 3]$
(c) $f'(x) \neq 3$ for any $x \in [1, 3]$
(d) $f'(x) \neq 5$ for any $x \in [1, 3]$

18. If $f(x) = x^3 + ax^2 + bx + c$, then

(a) $f(4) = 22$ (b) $f'(2) = 3$
(c) $f'(1) = 1$ (d) $f''(3) = 8$

Passage II

Directions (Q. Nos. 19-20) If $z = r(\cos\theta + i\sin\theta)$ and n is a positive integer, then

$z^{1/n} = r^{1/n}\left[\cos\frac{(2k\pi + \theta)}{n} + i\sin\frac{(2k\pi + \theta)}{n}\right]$, where

$k = 0, 1, 2, 3, \ldots (n - 1)$, gives n, nth roots of the complex number

19. If $1, \alpha_1, \alpha_2, \ldots, \alpha_{n-1}$ are the distinct nth roots of unity, then

(a) $(1 + \alpha_1)(1 + \alpha_2)\ldots(1 + \alpha_{n-1}) = \frac{1 + (-1)^n}{2}$

(b) $(1 + \alpha_1)(1 + \alpha_2)\ldots(1 + \alpha_{n-1}) = \frac{1 - (-1)^n}{2}$

(c) $\left|\cos\left(\frac{\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right)\cos\left(\frac{3\pi}{n}\right)\ldots\cos\left(\frac{(n-1)\pi}{n}\right)\right| = \frac{1 + (-1)^n}{2}$

(d) $\cos 4°\cos 8°\cos 12°\ldots\cos 88° = \frac{1}{2^{22}}$

20. If $1, \omega, \omega^2, \ldots, \omega^{n-1}$ are the n, nth root of unity and z_1 and z_2 are any two complex numbers, then $\sum_{k=0}^{n-1} |z_1 + \omega^k z_2|^2$ is equal to

(a) $n[|z_1|^2 + |z_2|^2]$
(b) $(n - 1)[|z_1|^2 + |z_2|^2]$
(c) $(n + 1)[|z_1|^2 + |z_2|^2]$
(d) $(n + 2)[|z_1|^2 + |z_2|^2]$

10
PRACTICE SETS
FOR JEE ADVANCED 2016

C200 ₹470

Think About it!

If the graph of $y = \sin \pi x$ is stretched vertically upward by a factor of 5 and is shifted 1/2 unit to the right, then the equation of function, whose graph is describe in words, is $y = 5\sin\left(\pi x - \frac{\pi}{2}\right)$. "Is it true".

Answers *with* Explanation

Paper 1

1. ***(7)*** We have,

$g(x) = x^2 - 2$

Let $g(x) = y$

$\therefore \quad y = x^2 - 2$

$\Rightarrow \quad x = \sqrt{y+2}$

Now, we have $p(x) = x^5 + x^2 + 1$

$\Rightarrow \quad (\sqrt{y+2})^5 + (\sqrt{y+2})^2 + 1 = 0$

$\Rightarrow \quad (y+2)^{5/2} = -(y+2+1)$

$\Rightarrow \quad (y+2)^{5/2} = -(y+3)$

On squaring both sides, we get

$(y+2)^5 = y^2 + 6y + 9$

$\Rightarrow \quad y^5 + 10y^4 + 40y^3 + 79y^2 + 74y + 23 = 0$

Now, product of roots $= -23$

i.e. $g(x_1)\cdot g(x_2)\cdot g(x_3)\cdot g(x_4)\cdot g(x_5) = -23$

and $x_1x_2x_3x_4x_5 = -1$

$\therefore \quad g(x_1)\cdot g(x_2)\cdot g(x_3)\cdot g(x_4)\cdot g(x_5) - 30g(x_1x_2x_3x_4x_5)$

$= -23 + 30 = 7$

2. ***(6)*** Let $\tan\frac{\pi}{8} = x$

$\Rightarrow \quad \frac{\pi}{8} = \tan^{-1}x$

$\Rightarrow \quad 4\tan^{-1}x = \frac{\pi}{2}$

$\Rightarrow \quad 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}\left[\frac{\left(\frac{4x}{1-x^2}\right)}{1-\left(\frac{2x}{1-x^2}\right)^2}\right] = \frac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}\left(\frac{4x(1-x^2)}{1-6x^2+x^4}\right) = \frac{\pi}{2}$

$\Rightarrow \quad x^4 - 6x^2 + 1 = 0$

Hence, the value of k is 6.

3. ***(1)*** We have,

$\cos y\left(\frac{dy}{dx} + e^{-x}\right) + \sin y\left(e^{-x} - \frac{dy}{dx}\right) = e^{e^{-x}}$

$\Rightarrow (\cos y - \sin y)\frac{dy}{dx} + e^{-x}(\cos y + \sin y) = e^{e^{-x}} \quad \ldots(i)$

Put $\cos y + \sin y = v$

$\Rightarrow \quad (\cos y - \sin y)\frac{dy}{dx} = \frac{dv}{dx}$

Eq. (i) reduces to,

$\frac{dv}{dx} + ve^{-x} = e^{e^{-x}}$

$\Rightarrow \quad v\cdot e^{-e^{-x}} = \int e^{e^{-x}}\cdot e^{-e^{-x}}\,dx$

$[\because \text{IF} = e^{\int e^{-x}dx} = e^{-e^{-x}}]$

$\Rightarrow \quad v\cdot e^{-e^{-x}} = x + C$

$\Rightarrow \quad (\cos y + \sin y)e^{-e^{-x}} = x + C$

$\Rightarrow \quad e^{-e^{-t}} = t \qquad [\text{putting } x = t, 0]$

$te^{e^{-t}} = 1$

4. ***(1)*** Solving the equation $zi - \bar{z}i + 2 = 0$

and $z(1+i) + \bar{z}(1-i) + 2 = 0$, we get

$z + \bar{z} = 0$

$\Rightarrow \quad z = -\bar{z}$

Now, on substituting $z = -\bar{z}$ in $zi - \bar{z}i + 2 = 0$, we get

$\because \quad z = i$

Distance between α and i is 2.

$\therefore |\alpha - i| = 2 \Rightarrow \alpha = i \pm 2e^{i\theta}$, put it in the second line, we get

$\cos\theta - \sin\theta = 0$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

$\therefore \quad \alpha = i \pm 2e^{i\pi/4}$

$\Rightarrow \quad \alpha = i \pm \left(2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)\right)$

$\Rightarrow \quad \alpha = \pm\sqrt{2} + i(1 \pm \sqrt{2})$

Hence, $x = \pm\sqrt{2}$

and $[|x|] = [|\sqrt{2}|] = 1$

5. ***(8)*** We have, 5 red and 4 blue points.

$\therefore$ Total number of points on circle $= (5 + 4) = 9$

	$n(S)$	$n(A)$
Triangle	${}^9C_3$	${}^9C_3 - {}^5C_3$
Quadrilateral	${}^9C_4$	${}^9C_4 - {}^5C_4$
Pentagon	${}^9C_5$	${}^9C_5 - {}^5C_5$
Hexagon	${}^9C_6$	${}^9C_6$
Heptagon	${}^9C_7$	${}^9C_7$
Octagon	${}^9C_8$	${}^9C_8$
Nonagon	${}^9C_9$	${}^9C_9$

Note that when 6 or more vertices are taken, then atleast one blue has to be taken

$\therefore \quad n(S) = 2^9 - ({}^9C_0 + {}^9C_1 + {}^9C_2) = 512 - (1 + 9 + 36)$

$n(S) = 512 - 46 = 466$

$n(A) = (84 - 10) + (126 - 5) + (126 - 1) + 84 + 36 + 3 + 1$

$= 74 + 121 + 125 + 84 + 36 + 3 + 1$

$= 450$

Required probability

$= \frac{n(A)}{n(S)} = \frac{450}{466} = \frac{225}{233}$

$\therefore \quad |n - m| = |233 - 225| = 8$

6. ***(5)*** We have,

$$A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix} \quad ...(i)$$

and $$A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix} \quad ...(ii)$$

Let $$A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$$

The first equation gives

$$a-b=-1 \quad ...(iii)$$

and $$c-d=2 \quad ...(iv)$$

Now, consider second equation, $A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

$$\Rightarrow \quad A\left(A\begin{bmatrix}1\\-1\end{bmatrix}\right)=\begin{bmatrix}1\\0\end{bmatrix}$$

$$\Rightarrow \quad A\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

This gives,

$$-a+2b=1 \quad ...(v)$$

and $$-c+2d=0 \quad ...(vi)$$

On solving Eqs. (iii) and (v), we get

$$a=-1$$

and $$b=0$$

and solving Eqs. (iv) and (vi), we get

$$c=4$$

and $$d=2$$

Thus, sum of elements

$$=-1+0+4+2=5$$

and the value of $k=5$

7. ***(8)*** We have,

$$1\cdot 0!+3\cdot 1!+7\cdot 2!+13\cdot 3!+\ldots+\text{ upto }(n+1)\text{ terms}$$

Clearly, $T_r=(r^2-r+1)(r-1)!$

$\Rightarrow \quad T_r=rr!-r!+(r-1)!$

$\Rightarrow \quad T_r=(r+1-1)r!-r!+(r-1)!$

$\Rightarrow \quad T_r=(r+1)!-2r!+(r-1)!$

$\Rightarrow \quad T_1=2!-2\cdot 1!+0!$

$T_2=3!-2\cdot 2!+1!$

$T_3=4!-2\cdot 3!+2!$

$T_n=(n+1)!-2(n)!+(n-1)!$

$T_{n+1}=(n+2)!-2(n+1)!+n!$

$$\Rightarrow \quad \sum_{r=1}^{n+1}T_r=(n+2)!-2(n+1)!+(n+1)!-2\cdot 1!+0!+1!$$

$$\Rightarrow \quad \sum_{r=1}^{n+1}T_r=(n+2)!-(n+1)!$$

$$=(n+1)![n+2-1]$$

$$=(n+1)(n+1)!$$

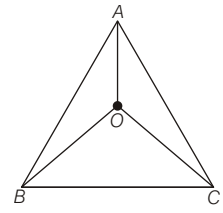
Given, $(n+1)(n+1)!=4000(4000)!$

$\therefore \quad n+1=4000$

and $$\frac{n+1}{500}=\frac{4000}{500}=8$$

8. ***(4)*** We have O is the interior point of ΔABC.

and $$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta AOC}=\frac{m}{n}$$



$$\Rightarrow \frac{\text{Area of }\Delta OAB+\text{ Area of }\Delta OBC+\text{ Area of }\Delta OAC}{\text{Area of }\Delta AOC}=\frac{m}{n}$$

$$\Rightarrow \quad \frac{\frac{1}{2}[|\mathbf{OA}\times\mathbf{OB}|+|\mathbf{OB}\times\mathbf{OC}|+|\mathbf{OA}\times\mathbf{OC}|]}{\frac{1}{2}|\mathbf{OA}\times\mathbf{OC}|}=\frac{m}{n}$$

$$\Rightarrow \quad \frac{|\mathbf{OA}\times\mathbf{OB}|+|\mathbf{OB}\times\mathbf{OC}|}{|\mathbf{OA}\times\mathbf{OC}|}+1=\frac{m}{n}$$

$$\Rightarrow \quad \frac{|\mathbf{OA}\times\mathbf{OB}|}{|\mathbf{OA}\times\mathbf{OC}|}+\frac{|\mathbf{OB}\times\mathbf{OC}|}{|\mathbf{OA}\times\mathbf{OC}|}+1=\frac{m}{n}$$

Now, as $\mathbf{OA}+2\mathbf{OB}+3\mathbf{OC}=\mathbf{0}$

$2\mathbf{OA}\times\mathbf{OB}+3\mathbf{OA}\times\mathbf{OC}=\mathbf{0}$

$$\Rightarrow \quad \frac{\mathbf{OA}\times\mathbf{OB}}{\mathbf{OA}\times\mathbf{OC}}=\frac{-3}{2}\Rightarrow\frac{|\mathbf{OA}\times\mathbf{OB}|}{|\mathbf{OA}\times\mathbf{OC}|}=\frac{3}{2} \quad ...(i)$$

Similarly,

$\mathbf{OB}\times\mathbf{OA}+3\mathbf{OB}\times\mathbf{OC}=\mathbf{0}$

$$\Rightarrow \quad \frac{\mathbf{OB}\times\mathbf{OA}}{\mathbf{OB}\times\mathbf{OC}}=-3\Rightarrow\frac{|\mathbf{OB}\times\mathbf{OA}|}{|\mathbf{OB}\times\mathbf{OC}|}=3 \quad ...(ii)$$

From Eqs. (i) and (ii), $\dfrac{|\mathbf{OB}\times\mathbf{OC}|}{|\mathbf{OA}\times\mathbf{OC}|}=\dfrac{1}{2}$

$\therefore \quad \frac{3}{2}+\frac{1}{2}+1=\frac{m}{n}\Rightarrow\frac{m}{n}=\frac{3}{1}$

$\Rightarrow \quad m+n=3+1=4$

9. ***(a,b,d)*** We have, $x^3+ax^2+bx+c=0$

Since, λ is a root of the equation.

Therefore, $$\lambda^3+a\lambda^2+b\lambda+c=0 \quad ...(i)$$

$\Rightarrow \quad \lambda^3=-a\lambda^2-b\lambda-c$

$\Rightarrow \quad \lambda^4=-a\lambda^3-b\lambda^2-c\lambda \quad ...(ii)$

On adding Eqs. (i) and (ii), we get

$\lambda^4=(1-a)\lambda^3+(a-b)\lambda^2+(b-c)\lambda+c$

$|\lambda|^4\le(1-a),|\lambda|^3+(a-b)$

$|\lambda|^2+(b-c),\quad|\lambda|+c \quad ...(i)$

If $|\lambda|\ge 1$, then $|\lambda|^4$

$\le(1-a)|\lambda|^3+(a-b)|\lambda|^3+(b-c)|\lambda|^3+c|\lambda|^3\le|\lambda|^3$

$\Rightarrow \quad |\lambda|\le 1$

Hence, the only possibility is $|\lambda|=1$.

Also for $|\lambda|<1$ above inequality satisfied.

If $|\lambda|<1$, then the inequality (i) hold good.

10. ***(b,c,d)*** We have, $f(x) = [x^{-2}[x^2]]$

We know that

$0 \le [x^2] \le x^2$

$\Rightarrow \quad 0 \le x^{-2}[x^2] \le x^{-2}x^2$

$\Rightarrow \quad 0 \le x^{-2}[x^2] \le 1$

$\therefore \quad f(x) = [x^{-2}[x^2]] = 0, 1$

$\therefore$ $f(x)$ is discontinuous at $x^2 = n, n \in N$

i.e. at $x = \sqrt{n}$ and hence also non-differentiable at these points.

11. ***(b,c)*** Let $\alpha = \hat{i} + x\hat{j} + 3\hat{k}$

and $\beta = 4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}$

Since, it is given that $2|\alpha| = |\beta|$

$\therefore \quad 2\sqrt{10 + x^2} = \sqrt{20 + 4(2x - 1)^2}$

On squaring both sides, we get

$4(10 + x^2) = 20 + 4(2x - 1)^2$

$\Rightarrow \quad 10 + x^2 = 5 + 4x^2 + 1 - 4x$

$\Rightarrow \quad 3x^2 - 4x - 4 = 0 \Rightarrow x = 2, -\frac{2}{3}$

12. ***(a,c)*** We have, $f(x) = x + \sin x$

and $I = \int_0^{\pi} f^{-1}(x)\,dx$

Put $f^{-1}(x) = t \Rightarrow x = f(t)$

$\Rightarrow \quad dx = f'(t)\,dt$

$\Rightarrow \quad I = \int_0^{\pi} t\,f'(t)\,dt$

$\Rightarrow \quad I = [t \cdot f(t)]_0^{\pi} - \int_0^{\pi} f(t)\,dt$

$\Rightarrow \quad I = (\pi\, f(\pi)) - \int_0^{\pi} f(x)\,dx \qquad [\because \int_0^a f(x)\,dx = \int_0^a f(t)\,dt]$

$\Rightarrow \quad I = \pi\,[\pi + \sin\,\pi] - \int_0^{\pi} (x + \sin x)\,dx$

$\Rightarrow \quad I = \pi^2 - \left[\frac{x^2}{2} - \cos\,x\right]_0^{\pi}$

$\Rightarrow \quad I = \frac{\pi^2}{2} - 2 \approx 2.89$

$\Rightarrow \quad 2 < I < 3$

Note that

$\frac{1}{2} < \int_0^1 \frac{1}{1 + x^3}\,dx < 1 \qquad [\because \frac{1}{1 + x^3}$ is decreasing function in $[0, 1]]$

Similarly, $\int_0^1 e^{x^2}\,dx$ lies between [1, e].

13. ***(b,c)*** Given,

$\sin^2 B \sin^2 C + 2\sin^2 C \sin^2 A + 2\sin^2 A \sin^2 B = \sin^4 A + \sin^4 B + \sin^4 C$

$\Rightarrow \quad \sin^4 A + \sin^4 B + \sin^4 C - 2\sin^2 C \sin^2 A - 2\sin^2 A \sin^2 B = \sin^2 B \sin^2 C$

$\Rightarrow \quad k^4a^4 + k^4b^4 + k^4c^4 - 2k^4a^2c^2 - 2k^4a^2b^2 = k^4b^2c^2$ [by sine rule]

$\Rightarrow \quad a^4 + b^4 + c^4 - 2a^2c^2 - 2a^2b^2 = b^2c^2$

On adding $2b^2c^2$ both sides, we get

$a^4 + b^4 + c^4 - 2a^2c^2 - 2a^2b^2 + 2b^2c^2 = 3b^2c^2$

$\Rightarrow \quad (b^2 + c^2 - a^2)^2 = 3b^2c^2$

$\Rightarrow \quad \left(\frac{b^2 + c^2 - a^2}{2bc}\right)^2 = \frac{3}{4}$

$\Rightarrow \quad \cos^2 A = \frac{3}{4}$

$\Rightarrow \quad \cos A = \pm\frac{\sqrt{3}}{2} \Rightarrow A = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

14. ***(a,b,c,d)*** The word 'EVOLUTION' has 4 vowels, i.e. (EOUI) and 4 consonants, i.e. (VLTN)

Case I When two O and 3 consonants

Number of words $= {}^4C_3\frac{5!}{2!} = 4 \times 60 = 240$

Case II When any two vowels from EOUI and 3 consonants

Number of words $= {}^4C_2 \cdot {}^4C_3 \times 5! = 6 \times 4 \times 120 = 2880$

Total numbers of words $= 240 + 2880 = 3120$

$\because \; 3120 = 2^4 \times 3^1 \times 5^1 \times 13^1$

$\because \; a = 4, b = 1, c = 1, d = 1$

15. ***(c,d)*** The equation of any plane through the given line is

$(x + 3y + 6) + \lambda(3x - y - 4z) = 0$

$\Rightarrow \quad (1 + 3\lambda)x + (3 - \lambda)y - 4\lambda z + 6 = 0$

If the distance of this plane from the origin is unity,

then $\frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + 16\lambda^2}} = 1$

$\Rightarrow \quad (1 + 3\lambda)^2 + (3 - \lambda)^2 + 16\lambda^2 = 36$

$\Rightarrow \quad 26\lambda^2 = 26 \Rightarrow \lambda = \pm 1$

Hence, the equations of the planes are

$2x + y - 2z + 3 = 0 \qquad [\because \lambda = 1]$

and $x - 2y - 2z - 3 = 0 \qquad [\because \lambda = -1]$

16. ***(a,b,c)*** The number $xyz\,xyz$ is written in the form

$100000x + 10000y + 1000z + 100x + 10y + z$

$= 1000(100x + 10y + z) + 1(100x + 10y + z)$

$= (100x + 10y + z)(1000 + 1)$

$= (100x + 10y + z)(1001)$

$= (100x + 10y + z)(7 \times 11 \times 13)$

$\therefore$ The number xyz, xyz is divisible by 7, 11 and 13.

17. ***(b,c)*** Let $I = \int_{\lambda}^{\lambda + \frac{\pi}{2}} (\cos^4 x + \sin^4 x)\,dx$

Since, period of $(\cos^4 x + \sin^4 x)$ is $\frac{\pi}{2}$.

$\therefore \; I = \int_{\lambda}^{\lambda + \frac{\pi}{2}} (\cos^4 x + \sin^4 x)\,dx = \int_0^{\frac{\pi}{2}} (\cos^4 x + \sin^4 x)\,dx$

$[\because$ if T is period of $f(x)$, then $\int_a^{a+T} f(x)$ is independent of $a]$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \cos^4 x\,dx + \int_0^{\frac{\pi}{2}} \sin^4 x\,dx$

$\Rightarrow \quad I = \left(\frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}\right) + \left(\frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}\right)$

$= 2 \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} = \frac{3\pi}{8}$

18. ***(a,c)*** We know that,

$A^3 + B^3 + C^3 - 3ABC = (A + B + C)(A^2 + B^2 + C^2 - AB - AC - BC)$

$\Rightarrow \quad A^3+B^3+C^3=(A+B+C)$
$(A^2+B^2+C^2-AB-AC-BC)$ [$\because ABC=0$]

$\Rightarrow \quad A^3+B^3+C^3=(A+B+C)(A+\omega B+\omega^2 C)$
$(A+\omega^2 B+\omega C)$
[$\because \omega$ is complex cube root of unity]

$\Rightarrow \quad A^3+B^3+C^3=(A+B+C)$
$(A+\omega B+\omega^2 C)\,\overline{(A+\omega B+\omega^2 C)}$

$\Rightarrow \det(A^3+B^3+C^3)\cdot\det(A+B+C)$
$=\det(A+B+C)^2\cdot\det(A+\omega B+\omega^2 C)$
$\cdot\det\overline{(A+\omega B+\omega^2 C)}$

$\Rightarrow \det(A^3+B^3+C^3)\cdot\det(A+B+C)$
$=(\det(A+B+C))^2\cdot(\det(A+\omega B+\omega^2 C))^2\geq 0$

Hence, $\lambda\geq 0$

19. **A → r, B → s, C → q, D → p**

A. We have, $\frac{x^2+5}{2}=x-2\cos(ax+b)$

$\Rightarrow 2\cos(ax+b)=\frac{2x-x^2-5}{2}$

$\Rightarrow 2\cos(ax+b)=\frac{1}{2}[-4-(x-1)^2]$

LHS ≥ -2 and RHS ≤ -2

Equality may be possible

$\because$ RHS will be -2, when $x=1$

$\therefore$ LHS $=\cos(ax+b)=-1$, when $x=1$

i.e. $\cos(a+b)=-1$

$\Rightarrow a+b=\pi, 3\pi, 5\pi$ because $a+b\leq 16$

Hence, maximum value is 5π.

B. We have, $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{2}{9}+\tan^{-1}\frac{4}{33}+\ldots$

Let $T_1=\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{1}{1+2}=\tan^{-1}2-\tan^{-1}1$,

$T_2=\tan^{-1}\frac{2}{9}=\tan^{-1}\frac{2}{1+8}=\tan^{-1}2^2-\tan^{-1}2$,

$T_3=\tan^{-1}\frac{4}{33}=\tan^{-1}\frac{2^3-2^2}{1+2^3\cdot 2^2}=\tan^{-1}2^3-\tan^{-1}2^2$,

$\vdots \quad \vdots$

$T_n=\tan^{-1}2^n-\tan^{-1}2^{n-1}$

Sum to n terms $=\tan^{-1}2^n-\tan^{-1}(1)$

Sum to infinity $=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

C. We have, $\sin^{-1}\sqrt{x^2+2x+1}+\sec^{-1}\sqrt{x^2+2x+1}=\frac{\pi}{2}$

This equation will satisfy, if

$\sqrt{(x+1)^2}=\frac{1}{\sqrt{(x+1)^2}}\Rightarrow (x+1)^2=1$

$\Rightarrow x+1=\pm 1\Rightarrow x=-2$ or 0

Solutions are $x=0$ or $x=-2$

But when $x=0$, $\sec^{-1}\left(\frac{x}{2}\right)$ is not defined

and when $x=-2$

$2\sec^{-1}\left(\frac{x}{2}\right)+\sin^{-1}\left(\frac{x}{2}\right)=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$

D. The given equation is $1+\sin^4 x=\cos^2 3x$

$\Rightarrow \sin^4 x+1-\cos^2 3x=0$

$\Rightarrow \sin^4 x+\sin^2 3x=0$

$\Rightarrow \sin^4 x+(3\sin x-4\sin^3 x)^2=0$

$\Rightarrow \sin^2 x(16\sin^4 x-23\sin^2 x+9)=0$

$\Rightarrow \sin^2 x=0$ or $16\sin^4 x-23\sin^2 x+9=0$

$\Rightarrow \sin x=0$ or $16\sin^4 x-23\sin^2 x+9$ has no real roots

$\Rightarrow \sin x=0$

Now, the greatest positive solution in the interval

$\left[\frac{-5\pi}{2},\frac{5\pi}{2}\right]$ is 2π.

20. **A → s, B → r, C → q, D → p**

A. For 3 distinct real numbers matrices formed are of the order $3\times 1, 1\times 3\Rightarrow$ number of possible matrices will be $3!+3!=6+6=12$.

B. To form a triangle three line segments are required, but it should also satisfy the condition that sum of lengths of any two sides should be greater than the third side.

Hence, required number of triangles

$={}^6C_3-7=13$, as we are excluding

(2, 3, 5), (2, 3, 6), (2, 3, 7), (2, 4, 6), (2, 4, 7), (2, 5, 7), (3, 4, 7)

C. Given, $\mathbf{x}+\mathbf{y}+\mathbf{z}=\mathbf{a}$

$\Rightarrow \quad \mathbf{a}\cdot\mathbf{x}+\mathbf{a}\cdot\mathbf{y}+\mathbf{a}\cdot\mathbf{z}=|\mathbf{a}|^2$

$\Rightarrow \quad \frac{3}{2}+\frac{7}{4}+\mathbf{a}\cdot\mathbf{z}=4$
[$\because \mathbf{a}\cdot\mathbf{x}=\frac{3}{2}, \mathbf{a}\cdot\mathbf{y}=\frac{7}{4}, |\mathbf{a}|=2$]

$\Rightarrow \quad \mathbf{a}\cdot\mathbf{z}=\frac{3}{4}$

Also, from given equation, we get

$\mathbf{x}\cdot\mathbf{y}+\mathbf{x}\cdot\mathbf{z}=\frac{1}{2}$ ...(i)

$\mathbf{x}\cdot\mathbf{y}+\mathbf{y}\cdot\mathbf{z}=\frac{3}{4}$ ...(ii)

$\mathbf{x}\cdot\mathbf{z}+\mathbf{y}\cdot\mathbf{z}=\frac{1}{4}$ ...(iii)

On solving Eqs. (i), (ii) and (iii), we get

$\mathbf{y}\cdot\mathbf{z}=0, \mathbf{z}\cdot\mathbf{x}=-\frac{1}{4}, \mathbf{x}\cdot\mathbf{y}=\frac{3}{4}$

Now, consider $(\mathbf{x}\times\mathbf{y})\times\mathbf{z}=\mathbf{c}\Rightarrow(\mathbf{x}\cdot\mathbf{z})\mathbf{y}-(\mathbf{y}\cdot\mathbf{z})\mathbf{x}=\mathbf{c}$

$\Rightarrow \quad -\frac{1}{4}\mathbf{y}=\mathbf{c}\Rightarrow\mathbf{y}=-4\mathbf{c}\Rightarrow\lambda=4\Rightarrow 2\lambda=8$

D. Consider $\sin 1^\circ=\sin((x+1)^\circ-x^\circ)=\sin(x+1)^\circ\cos(x)^\circ-\cos(x+1)^\circ\sin x^\circ$

$\Rightarrow \quad \frac{\sin 1^\circ}{\sin x^\circ\sin(x+1)^\circ}=\cot x^\circ-\cot(x+1)^\circ$

$\because \quad \frac{1}{\sin 45^\circ\sin 46^\circ}+\frac{1}{\sin 46^\circ\sin 47^\circ}+\ldots+\frac{1}{\sin 134^\circ\sin 135^\circ}=\frac{n}{\sin 1^\circ}$

$\therefore \quad [\cot 45^\circ-\cot 46^\circ+\cot 46^\circ-\cot 47^\circ\ldots+\cot 134^\circ-\cot 135^\circ]=\frac{\sin 1^\circ}{\sin 1^\circ}\cdot n$

$\Rightarrow \quad (\cot 45^\circ-\cot 135^\circ)=n$

$\Rightarrow \quad 2=n \quad \Rightarrow \quad 3n=6$

Paper 2

1. ***(4)*** Clearly, $z^5 - 32 = (z - z_0)(z - z_1)(z - z_2)(z - z_3)(z - z_4)$,

where $z_i = 2\left(\cos\frac{2m\pi}{5} + i\sin\frac{2m\pi}{5}\right)$

(using De-Moivre's theorem)

and $i = 0, 1, 2, 3, 4$ with $m = 0, 1, 2, 3, 4$

Now, $z_0 = 2$

$z_1 = 2\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$, $z_2 = 2\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right)$,

$z_3 = 2\left(\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}\right)$, $z_4 = 2\left(\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}\right)$

Hence, $z^5 - 32 = (z-2)\left(z^2 - 4\cos\frac{2\pi}{5}z + 4\right)\left(z^2 - 4\cos\frac{4\pi}{5} + z\right)$

$\therefore p = 4\cos\frac{2\pi}{5} = 4\cos 72° = 4\sin 18°$

and $p^2 + 2p = 16\sin^2 18° + 8\sin 18°$

$= 8[2\sin^2 18° + \sin 18°]$

$= 8[1 - \cos 36° + \sin 18°]$

$= 8\left[1 - \left(\frac{\sqrt{5}+1}{4}\right) + \left(\frac{\sqrt{5}-1}{4}\right)\right] = 8\left[\frac{1}{2}\right] = 4$

2. ***(3)*** We know that the product of length of perpendicular drawn from foci to any tangent of ellipse is b^2, i.e. $SP \times S'P' = b^2$

$\therefore \sum_{i=1}^{10}(SP_i \times S'P_i') = \sum_{i=1}^{10} b^2 = 10b^2 = 2560 \Rightarrow b^2 = 256$

As, $a = 20$, therefore $a^2 = 400$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{256}{400}} = \frac{3}{5} \Rightarrow 5e = 5 \times \frac{3}{5} = 3$

3. ***(4)*** Since, $P(x) = x^2 + bx + c$, divides both of them.

Therefore, $P(x)$ also divides

$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$

$= -14x^2 + 28x - 70$

$= -14(x^2 - 2x + 5)$, which is a quadratic equation.

Hence, $P(x) = x^2 - 2x + 5$

$\therefore P(1) = 1 - 2 + 5 = 4$

4. ***(0)*** We have, $f(x) = \int_0^x \sin(t^2 - t + x)\,dt$

$\Rightarrow f(x) = \int_0^x [\sin(t^2 - t)\cos x + \sin x\cos(t^2 - t)]\,dt$

$\Rightarrow f'(x) = -\sin x\int_0^x \sin(t^2 - t)\,dt + \cos x\sin(x^2 - x) + \cos x\int_0^x \cos(t^2 - t)\,dt + \sin x\cos(x^2 - x)$

$\Rightarrow f''(x) = -\sin x\sin(x^2 - x) - \cos x\int_0^x \sin(t^2 - t)\,dt + \cos x\cos(x^2 - x)(2x - 1) - \sin x\sin(x^2 - x) + \cos x\cos(x^2 - x) - \sin x\int_0^x \cos(t^2 - t)\,dt + \cos x\cos(x^2 - x) - (2x - 1)\sin(x^2 - x)\sin x$

$\Rightarrow f''(x) + f(x) = (2x + 1)\cos x^2$

$\Rightarrow f''\left(-\frac{1}{2}\right) + f\left(-\frac{1}{2}\right) = (-1 + 1)\cos\frac{1}{4} = 0$

5. ***(3)*** Given, ${}^9C_n + 3\cdot{}^9C_{n+1} + 3\cdot{}^9C_{n+2} + {}^9C_{n+3} > {}^{12}C_{n+2}$

$\Rightarrow {}^9C_n + {}^9C_{n+1} + 2\cdot{}^9C_{n+1} + 2\cdot{}^9C_{n+2} + {}^9C_{n+2} + {}^9C_{n+3} > {}^{12}C_{n+2}$

$\Rightarrow {}^{10}C_{n+1} + 2({}^{10}C_{n+2}) + {}^{10}C_{n+3} > {}^{12}C_{n+2}$

$\Rightarrow {}^{10}C_{n+1} + {}^{10}C_{n+2} + {}^{10}C_{n+2} + {}^{10}C_{n+3} > {}^{12}C_{n+2}$

$\Rightarrow {}^{11}C_{n+2} + {}^{11}C_{n+3} > {}^{12}C_{n+2}$

$\Rightarrow {}^{12}C_{n+3} > {}^{12}C_{n+2}$

$\Rightarrow \frac{12!}{(n+3)!(9-n)!} > \frac{12!}{(n+2)!(10-n)!}$

$\Rightarrow 10 - n > n + 3 \Rightarrow n < \frac{7}{2}$

Since, n is a natural number.

$\therefore n = 3$

6. ***(9)*** We have, $A^2 - 5A + 7I = 0$

$\Rightarrow A^2 = 5A - 7I$

$\Rightarrow A^4 = (5A - 7I)^2 = 25A^2 - 70A + 49I$

$\Rightarrow A^4 = 25(5A - 7I) - 70A + 49I = 55A - 126I$

$\Rightarrow A^6 = A^4 \cdot A^2 = (55A - 126I)(5A - 7I)$

$\Rightarrow A^6 = 275A^2 - 385A - 630A + 882I$

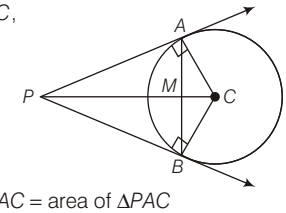
$\Rightarrow A^6 = 275(5A - 7I) - 1015A + 882I$

$= 360A - 1043I$

$\Rightarrow a = 360$ and $b = -1043$

$\therefore \left[\frac{a-b}{150}\right] = \left[\frac{1403}{150}\right] = 9$

7. ***(8)*** In ΔPAC,



area of ΔPAC = area of ΔPAC

$\Rightarrow \frac{1}{2} \times PA \times AC = \frac{1}{2} \times PC \times AM$

$\Rightarrow \frac{1}{AM} = \frac{PC}{PA \times AC} \Rightarrow \frac{1}{AM^2} = \frac{PC^2}{PA^2 \times AC^2}$

$\Rightarrow \frac{1}{AM^2} = \frac{PA^2 + AC^2}{PA^2 \times AC^2} \Rightarrow \frac{1}{AM^2} = \frac{1}{AC^2} + \frac{1}{PA^2}$

$\therefore AM^2 = 16 \Rightarrow AM = 4 \quad \left[\because \frac{1}{CA^2} + \frac{1}{PA^2} = \frac{1}{16}\right]$

$\Rightarrow AB = 2AM = 2 \times 4 = 8$

8. ***(2)*** We have, $\lim_{x\to 0}\frac{1 + a^3 + 8e^{1/x}}{1 + (2 + b + b^2)e^{1/x}} = 2$

RHL $= \lim_{x\to 0^+}\frac{1 + a^3 + 8e^{1/x}}{1 + (2 + b + b^2)e^{1/x}} = 2$

$\Rightarrow 2 + b + b^2 = 4 \Rightarrow b^2 + b - 2 = 0 \Rightarrow b = 1$

LHL $= \lim_{x\to 0^-}\frac{1 + a^3 + 8e^{1/x}}{1 + (2 + b + b^2)e^{1/x}} = 2$

$\Rightarrow 1 + a^3 = 2$

$\Rightarrow a = 1$

$\therefore a^2 + b^2 = 1^2 + 1^2 = 2$

9. ***(b,d)*** Equation of tangent at (l, m) on the parabola $y^2 = 4ax$ is

$$my = 2a(x + l) \quad \ldots(i)$$

Any point on Eq. (i) is $\left(h, \frac{2a}{m}(h + l)\right)$

Equation of chord of contact to the circle $x^2 + y^2 = a^2$ is

$$hx + \frac{2a(h + l)}{m} y = a^2$$

$\Rightarrow \quad mhx + 2ahy + 2aly - ma^2 = 0$

$\Rightarrow \quad h(mx + 2ay) + 2aly - ma^2 = 0$

The fixed point is $\left(\frac{-a^2}{l}, \frac{ma}{2l}\right)$.

$\therefore \quad \alpha = \frac{-a^2}{l}$ and $\beta = \frac{ma}{2l} \Rightarrow \alpha \cdot l = -a^2$ and $\beta m = \frac{2\beta^2 l}{a}$

Clearly $\alpha l < 0$ and $\beta m > 0$

$\therefore$ Also, $\left[\frac{1}{2}\left(\frac{m}{\beta}\right)\right]^2 + \frac{l}{\alpha} = 0 \Rightarrow \frac{4l}{\alpha} + \left(\frac{m}{\beta}\right)^2 = 0$

10. ***(b,c)*** Clearly, from Descarte's rule of sign equation has 4 positive real roots, lets say α, β, γ and δ.

$\Rightarrow \Sigma\alpha = -a_1, \Sigma\alpha\beta = a_2, \Sigma\alpha\beta\gamma = -a_3$ and $\alpha\beta\gamma\delta = 4$

Using AM $\geq$ GM, we have

$$\left(\frac{\alpha + \beta + \gamma + \delta}{4}\right)\left(\frac{\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta}{4}\right) \geq \sqrt[4]{\alpha\beta\gamma\delta}\ \sqrt[4]{\alpha^3\beta^3\gamma^3\delta^3}$$

$\Rightarrow \quad \frac{-a_1}{4} \times \frac{-a_3}{4} \geq 4$

$\Rightarrow \quad a_1 a_3 \geq 64$

and $\frac{\alpha\beta + \beta\gamma + \alpha\gamma + \alpha\delta + \beta\delta + \gamma\delta}{6} \geq \sqrt[6]{\alpha^3\beta^3\gamma^3\delta^3}$

$\Rightarrow \quad \frac{a_2}{6} \geq (\alpha\beta\gamma\delta)^{3/6}$

$\Rightarrow \quad a_2 \geq 6(4)^{1/2}$

$\Rightarrow \quad a_2 \geq 12$

11. ***(a,b,c)*** (a) We have, $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

We know that, $|A - \lambda I| = 0$

$$\therefore \quad \begin{vmatrix} 1-\lambda & 0 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (1 - \lambda)(\lambda^2 - 1) = 0$

$\Rightarrow \lambda^2 - 1 - \lambda^3 + \lambda = 0 \Rightarrow \lambda^3 - \lambda^2 = \lambda - 1$

Hence, $A^3 - A^2 = A - I$

(b) $\det(A^{2016} - I) = \det(A - I) \det(1 + A + A^2 + A^3 + \ldots A^{2015})$

$\Rightarrow \det(A^{2016} - I) = 0 \times \det(1 + A + A^2 + \ldots + A^{2015}) = 0$ [$\because \det(A - I) = 0$]

(c) $\because \quad A^3 - A^2 = A - I \quad \ldots(i)$

$\therefore \quad A^4 - A^3 = A^2 - A \quad \ldots(ii)$

and $A^5 - A^4 = A^3 - A^2 = A - I$ [using Eq. (i)]

If n is even, then $A^n - A^{n-1} = A^2 - A \quad \ldots(iii)$

If n is odd, then $A^n - A^{n-1} = A - I \quad \ldots(iv)$

Consider n is even,

$A^n - A^{n-1} = A^2 - A$ ($\because n$ is even)

$A^{n-1} - A^{n-2} = A - I$ ($\because n - 1$ is odd)

$\Rightarrow \quad A^n - A^{n-2} = A^2 - I$

$\Rightarrow \quad A^n = A^{n-2} + A^2 - I$

$\Rightarrow \quad A^n = (A^{n-4} + A^2 - I) + A^2 - I$

$= (A^{n-6} + A^2 - I) + 2(A^2 - I)$

Similarly, $A^n = I + \frac{n}{2}(A^2 - I)\ 0 = \frac{n}{2}A^2 - \left(\frac{n-2}{2}\right)I$

$$\therefore \quad A^{2016} = 1008A^2 - 1007I = \begin{bmatrix} 1 & 0 & 0 \\ 1008 & 1 & 0 \\ 1008 & 0 & 1 \end{bmatrix}$$

12. ***(a,c,d)*** Given, $f(x) + \frac{3}{2} f\left(\frac{2x + 29}{x - 2}\right) = 50x + 40 \quad \ldots(i)$

Replacing x by $\frac{2x + 29}{x - 2}$ in Eq. (i), we get

$$f\left(\frac{2x + 29}{x - 2}\right) + \frac{3}{2} f(x) = \frac{50(2x + 29)}{x - 2} + 40 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$f(x) = 136 - 40x + \frac{1980}{x - 2}$$

$\therefore \quad A = 136, B = 40$ and $C = 396$

13. ***(b,c,d)*** Consider,

$$\sqrt{\sin^4 x + 4\cos^2 x} + \sqrt{\cos^4 x + 4\sin^2 x}$$

$$= \sqrt{\sin^4 x - 4\sin^2 x + 4} + \sqrt{\cos^4 x + 4 - 4\cos^2 x}$$

$= 2 - \sin^2 x + 2 - \cos^2 x = 3$

$\therefore$ Given equation reduces to

$\sqrt{1 + \cos 2x} = \sqrt{2}\ \sin^{-1}(\sin 2x)$

$|\cos x| = \sin^{-1}(\sin 2x)$

Now, let $f(x) = |\cos x| - \sin^{-1}(\sin 2x)$

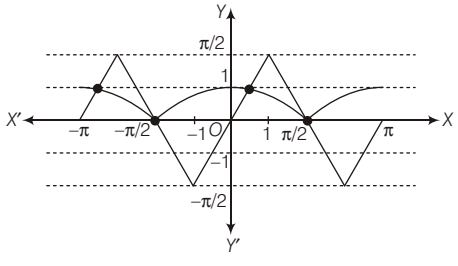
$\Rightarrow \quad f\left(\frac{\pi}{8}\right) = \left|\cos\frac{\pi}{8}\right| - \sin^{-1}\sin\frac{\pi}{4}$

$\Rightarrow \quad f\left(\frac{\pi}{8}\right) = \cos\frac{\pi}{8} - \frac{\pi}{4} = \sqrt{\frac{1 + \cos\frac{\pi}{4}}{2}} - \frac{\pi}{4} > 0$

and $\quad f\left(\frac{\pi}{6}\right) = \left|\cos\frac{\pi}{6}\right| - \frac{\pi}{3} = \frac{\sqrt{3}}{2} - \frac{\pi}{3} < 0$

So, one root exists between $\left(\frac{\pi}{8}, \frac{\pi}{6}\right)$.

Graph of $|\cos x| = \sin^{-1}\sin 2x$ in $[-\pi, \pi]$ is



14. ***(a,d)*** We have,

$f(x) - \mu\int_0^{\pi/4} \sin^2 x \cos^{-5} x \cos t\ f(t)\,dt = \sin^2 x \cos^{-5} x$

Let $\int_0^{\pi/4} \cos t\ f(t)\,dt = A$

$\therefore \quad f(x) = -\mu \sin^2 x \cos^{-5} x\ A = \sin^2 x \cos^{-5} x$

$\Rightarrow \quad f(x) = (\mu A + 1)\sin^2 x \cos^{-5} x$

$\Rightarrow \quad f(t) = (\mu A + 1)\sin^2 t \cos^{-5} t$

$\therefore \quad A = \int_0^{\pi/4} \cos t\ f(t)\,dt$

$\Rightarrow \quad A = (\mu A + 1)\int_0^{\pi/4} \frac{\sin^2 t}{\cos^4 t}\,dt$

$\Rightarrow \quad A = (\mu A + 1)\left[\frac{\tan^3 t}{3}\right]_0^{\pi/4} = (\mu A + 1)\frac{1}{3}$

$\Rightarrow \quad 3A = \mu A + 1 \Rightarrow A = \frac{1}{3-\mu}$

Hence, $f(x) = \left(\frac{\mu}{3-\mu} + 1\right)\sin^2 x \cos^{-5} x$

$\Rightarrow \quad f(x) = \frac{3}{3-\mu}\sin^2 x \cos^{-5} x$

So, period of $f(x)$ is 2π.

15. ***(a,c)*** We have, $\int_a^b \text{sgn}\,(x)\,dx = \int_a^b \frac{|x|}{x}\,dx \quad \left[\because \text{sgn}\,(x) = \frac{|x|}{x}\right]$

$= \left[\frac{|x|}{x}\int dx\right]_a^b - \int_a^b \frac{d}{dx}\text{sgn}\,(x)\cdot\int dx\cdot dx$

$= \left[\frac{|x|}{x}\cdot x\right]_a^b - 0 \qquad \left[\because \frac{d}{dx}\text{sgn}(x) = 0\right]$

$= [\,|x|\,]_a^b = |b| - |a|$

$= b\ \text{sgn}\ b - a\ \text{sgn}\ a \qquad [\because |x| = x\ \text{sgn}\,(x)]$

16. ***(a,b,c)*** We have, $(n-a)! = (n-b)!$.

Note that $(n-a)! = (n-b)!$ is possible iff

$n - a = 1$ and $n - b = 0$

$\therefore \quad a = n - 1$ and $b = n$

$\Rightarrow \quad 2a + b + 1 = 2n - 1 = k_1 \cdot n + k_2$

$\Rightarrow \quad k_1 = 2$ and $k_2 = -1$

So, $y = \log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) + \log_2\sqrt{4x^2 - 4x + 1} \Rightarrow y = 1$

The equation of circle is $x^2 + y^2 = 10$.

So, $P = (3, 1)$ whereas $Q \equiv (1, 1)$ or $(2, 1)$

$\Rightarrow \quad \mathbf{OP}\cdot\mathbf{OQ} = 3 + 1 = 4$ or $6 + 1 = 7$

and $\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP} = -i$ or $-2i$

$\Rightarrow \quad |\mathbf{PQ}| = 1$ or 2

17. ***(b)*** We have, $f : R \to R$ and $f(1) = 1$, $f(2) = 4$ and $f(3) = 9$

$\therefore \quad f(x) - x^2 = (x-1)(x-2)(x-3)$

$\Rightarrow \quad f(x) = x^2 + (x-1)(x-2)(x-3)$

$\Rightarrow \quad f'(x) = 2x + (x-1)(x-2) + (x-1)(x-3) + (x-3) + (x-2)(x-3)$

$\Rightarrow \quad f''(x) = 2 + (x-1) + (x-2) + (x-1) + (x-3) + (x-2) + (x-3)$

$\Rightarrow \quad f''(x) = 6x - 10$

$\Rightarrow \quad f''(2) = 12 - 10 = 2$

Hence, $f''(x) = 2$ for some $x \in [1, 3]$

18. ***(a,b,d)*** We have,

$f(x) = x^2 + (x-1)(x-2)(x-3)$

$f(4) = 16 + (4-1)(4-2)(4-3) = 16 + 6 = 22$

$f'(x) = 2x + (x-1)(x-2) + (x-1)(x-3) + (x-2)(x-3)$

$f'(2) = 4 + 0 + (2-1)(2-3) + 0 = 4 - 1 = 3$

$f'(1) = 2 + 0 + 0 + (1-2)(1-3) = 2 + 2 = 4$

$f''(x) = 6x - 10$

$f''(3) = 6 \times 3 - 10 = 18 - 10 = 8$

19. ***(b,d)*** Given, $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}$ are n distinct, nth roots of unity.

$\therefore \quad (z-1)(z-\alpha_1)(z-\alpha_2)\ldots(z-\alpha_{n-1}) = z^n - 1$

Putting $z = -1$, we get

$$(1+\alpha_1)(1+\alpha_2)\ldots(1+\alpha_{n-1}) = \frac{1-(-1)^n}{2}$$

Now, $|1 + \alpha_n| = \left|1 + \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}\right| = 2\left|\cos\frac{k\pi}{n}\right|$

For $n = 45$, $|1 + \alpha_n| = 2\left|\cos\frac{k\pi}{45}\right| = 2|\cos(4k)|$

as $\prod_{k=1}^{44}(1 + \alpha_k) = \frac{1-(-1)^{45}}{2} = 1$

$\Rightarrow 2^{44}(\cos 4^\circ \cos 8^\circ \cos 12^\circ \ldots \cos 176^\circ) = 1$

$\Rightarrow \cos 4^\circ \cos 8^\circ \cos 12^\circ \ldots \cos 88^\circ = \frac{1}{2^{22}}$

20. ***(a)*** We have,

$$\sum_{k=0}^{n-1}|z_1 + \omega^k z_2|^2 = \sum_{k=0}^{n-1}(z_1 + \omega^k z_2)(\bar{z}_1 + \bar{\omega}^k\bar{z}_2)$$

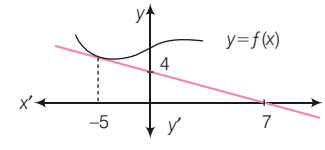
$= \sum_{k=0}^{n-1}(z_1\bar{z}_1 + (\omega\bar{\omega})^k\ z_2\bar{z}_2 + \bar{z}_1 z_2\omega^k + z_1\bar{z}_2\bar{\omega}^k)$

$= \sum_{k=0}^{n-1}(|z_1|^2 + |z_2|^2) + (z_2\bar{z}_1)\frac{(1-\omega^n)}{1-\omega} + z_1\bar{z}_2\left(\frac{1-\bar{\omega}^n}{1-\bar{\omega}}\right)$

$= n[|z_1|^2 + |z_2|^2]$

Think About it!

In the given figure, if red line is tangent to the graph of $y = f(x)$ at the indicated point, then



$$f(-5) = \frac{48}{7}$$

"It is true".

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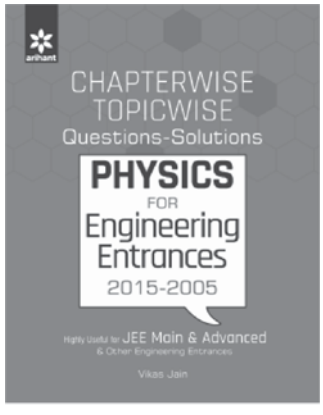

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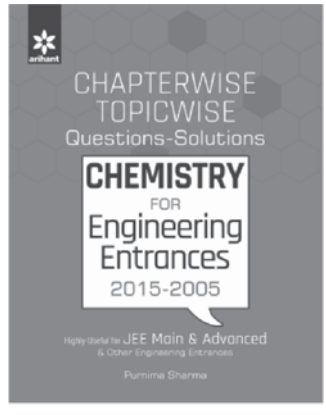

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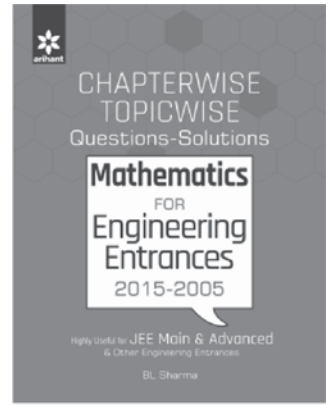

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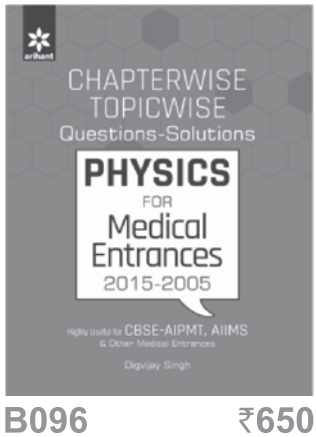

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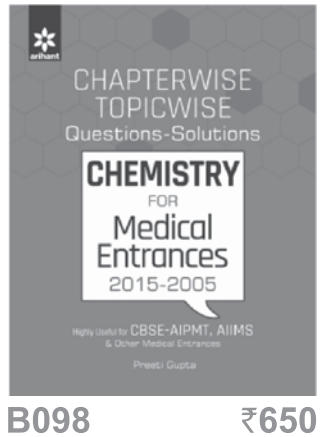

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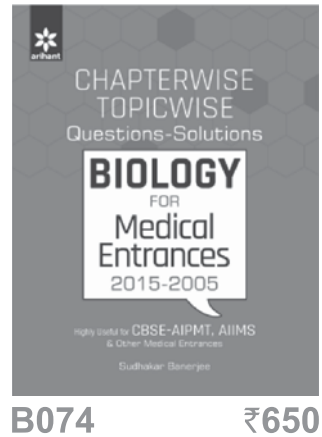

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Paper 1

Section I

Single Digit Integer Type

1. $f : R \to R$ is a twice differentiable function satisfying $f''(x) - 5f'(x) + 6f(x) \geq 0, \forall x \geq 0$, with $f(0) = 1$ and $f'(0) = 0$. If $f(x)$ satisfies $f(x) \geq ah(bx) - bh(ax), \forall x \geq 0$, then find the value of $(a + b)\, h(0)$.

2. If non-zero complex numbers z_1, z_2, z_3 represents the vertices of a triangle with the corresponding opposite sides being in the ratio $2 : 1 : \sqrt{3}$, then find the value of $\dfrac{4z_1^2 + z_2^2 + 3z_3^2}{z_1 (z_2 + 3z_3)}$.

3. If $5a + 4b + 20c = 5t$, then find the value of t for which the line $ax + by + c - 1 = 0$ always passes through a fixed point.

4. If the value of $\tan^6 40° - 33 \tan^4 40° + 27 \tan^2 40°$ is k, then k is equal to

5. If A is an idempotent matrix (non-zero) and I is an identity matrix of the same order. Find the value of $n, n \in N$, such that $(A + I)^n = I + 127\, A$.

6. If $(1 + x + x^2)^{3n+1} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{6n+2} x^{6n+2}$, then the value of $\displaystyle\sum_{r=0}^{2n} \left\{ a_{3r} - \left(\frac{a_{3r+1} + a_{3r+2}}{2} \right) \right\}$ is

7. In a five-team tournament, each team plays one game with every other team. Each team has 50% chance of winning any game that it plays. (there are no ties). If the probability that the tournament will produce neither an undefeated team nor a win less team is $\dfrac{m}{n}$, then the value of $\left| \dfrac{n-m}{3} \right|$ is

8. A square $PQRS$ of side length p is folded along the diagonal PR so that the point Q reaches at Q' and planes PRQ' and PRS are perpendicular to one another. The shortest distance between PQ' and RS is $\dfrac{kp}{\sqrt{6}}$, then the value of k is

Section II

One or More than One Options Correct Type

9. If z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then

(a) $3 \leq |z_1 - 2z_2| \leq 5$

(b) $1 \leq |z_1 + z_2| \leq 3$

(c) $|z_1 - 3z_2| \geq 5$

(d) $|z_1 - z_2| \geq 1$

10. $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are three coplanar unit vectors such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$. If three vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ parallel to $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ respectively and having integral but different magnitudes, then among the following options $|\mathbf{p} + \mathbf{q} + \mathbf{r}|$ can take a value equal to

(a) 1 *(b)* 0 *(c)* $\sqrt{3}$ *(d)* 2

11. If $f(x) = \dfrac{[x] + 1}{\{x\} + 1}$ for $f : \left[0, \dfrac{5}{2}\right) \to \left(\dfrac{1}{2}, 3\right]$, where $[\cdot]$ represents greatest integer function and $\{\cdot\}$ represents fractional part function, then which of the following is true?

(a) $f(x)$ is injective discontinuous function

(b) $f(x)$ is surjective non-differentiable function

(c) $\min\left(\lim\limits_{x \to 1^-} f(x), \lim\limits_{x \to 1^+} f(x) \right) = f(1)$

(d) max (x values of point of discontinuity) = $f(1)$

12. If $I = \int_0^1 \frac{dx}{1 + x^{\pi/2}}$, then

(a) $I > \ln 2$ (b) $I < \ln 2$
(c) $I < \pi/4$ (d) $I > \pi/4$

13. Two persons A and B have $n+1$ and n coins respectively, which they toss simultaneously. Then, the probability that A will have more heads than B is

(a) $\frac{1}{2}$ (b) $> \frac{1}{2}$ (c) $< \frac{1}{2}$ (d) $> \frac{1}{3}$

14. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

15. The function $f(x) = \cos^{-1}\left(\frac{2[|\sin x| + |\cos x|]}{\sin^2 x + 2\sin x + \frac{11}{4}}\right)$ is defined, if x belongs to (where, [.] represents greatest integer function)

(a) $\left[0, \frac{7\pi}{6}\right]$ (b) $\left[0, \frac{\pi}{6}\right]$ (c) $\left[\frac{11\pi}{6}, 2\pi\right]$ (d) $[\pi, 2\pi]$

16. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{\lambda}$ and $\frac{x-1}{\lambda} = \frac{y-4}{2} = \frac{z-5}{1}$ intersect at a point (α, β, γ), then

(a) $\lambda = -1$ (b) $\lambda = 2$ (c) $\lambda = -3$ (d) $\lambda = 0$

17. Let ABC be an isosceles triangle with base BC. If r is the radius of the circle inscribed in the ΔABC and ρ be the radius of the circle escribed opposite to the angle A, then the product of ρr can be equal to

(a) $R^2 \sin^2 A$ (b) $R^2 \sin^2 2B$ (c) $\frac{a^2}{2}$ (d) $\frac{a^2}{4}$

18. If the derivative of an odd cubic polynomial vanishes at two different values of x, then

(a) coefficient of x^3 and x in the polynomial must be of same sign

(b) coefficient of x^3 and x in the polynomial must be different in sign

(c) the value of x, where derivative vanishes, are closer to origin as compared to the respective roots on either side of origin

(d) the value of x, where derivative vanishes, are far from origin as compared to the respective roots on either side of origin

Section III

Match the Columns Type

19. Match the statements given in Column I with the values of Column II.

	Column I		Column II
A.	One ball is drawn from a bag containing 4 balls and is found to be white. The events that the bag contains '1 white', '2 white', '3 white' and '4 white' are equally likely. If the probability that all the white is $\frac{m}{n}$, then $(m + n)$ is equal to	p.	6
B.	A seven digit number made up of all distinct digits 8, 7, 6, 4, 2, x and y is divisible by 3. Then, possible number of ordered pair (x, y) is	q.	7
C.	If $x, y, z > 0$ and $x + y + z = 1$, then the least value of $\frac{5x}{2-x} + \frac{5y}{2-y} + \frac{5z}{2-z}$ is	r.	8
D.	If $\sum_{k=1}^{12} 12k\,{}^{12}C_k \cdot {}^{11}C_{k-1}$ is equal to $\frac{12 \times 21 \times 19 \times 17 \times 15 \times \ldots \times 3}{11!} \times 2^{12} \times p$, then the value of p is	s.	3

20. Match the statements given in Column I with the values given in Column II.

	Column I		Column II
A.	If $\mathbf{a} = \hat{\mathbf{j}} + \sqrt{3}\,\hat{\mathbf{k}}$, $\mathbf{b} = -\hat{\mathbf{j}} + \sqrt{3}\,\hat{\mathbf{k}}$ and $\mathbf{c} = 2\sqrt{3}\hat{\mathbf{k}}$ form a triangle, then the internal angle of the triangle between $\mathbf{a}$ and $\mathbf{b}$ is	p.	π
B.	If $\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	q.	$\frac{2\pi}{3}$
C.	The value of $\frac{\pi^2}{\log 3}\int_{7/6}^{5/6} \sec(\pi x)\,dx$ is	r.	$\frac{\pi}{2}$
D.	The maximum value of $\left\lvert \arg\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert = 1$, $z \neq 1$ is given by	s.	$\frac{\pi}{6}$

Paper 2

Section I

Single Digit Integer Type

1. If $f(x)$ is a polynomial of degree four with leading coefficient one satisfying $f(1)=1, f(2)=2, f(3)=3$, then $\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right]$ (where, $[\cdot]$ represents greatest integer function) is equal to

2. The number of solutions of the equation $e^{-\sqrt{\log x}} - \{x\}^{1/\sqrt{|\log x|}} = [\text{sgn}(x)]$ (where, $[\cdot]$ is greatest integer function and $\{.\}$ is fractional part function) is /are

3. If the line $y=\sqrt{3}x$ cuts the curve $x^4 + ax^2y + bxy + cx + dy + 6 = 0$ at A, B, C and D, then the value of $\frac{1}{12} OA \cdot OB \cdot OC \cdot OD$ (where, O is the origin) is

4. If ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots + (-1)^r {}^nC_r = 28$, then n is equal to

5. If $I_1 = \int_0^{\pi} \frac{\sin 884x \sin 1122x}{\sin x}\,dx$ and

$I_2 = \int_0^1 \frac{x^{238}(x^{1768}-1)}{x^2-1}\,dx$, then the value of $\frac{I_1}{I_2}$ is

6. Let S_1 and S_2 denote the circles

$x^2 + y^2 + 10x - 24y - 87 = 0$ and

$x^2 + y^2 - 10x - 24y + 153 = 0$, respectively. (Let m be the smallest positive value of a for which the line $y = ax$ contains the centre of a circle which touches S_2 externally and S_1 internally). Given that $m^2 = p/q$, where p and q are relatively prime integers. If $(p+q)$ is equal to 13^k, then the value of k is

7. If $f(x+1) = (-1)^{x+1}x - 2f(x)$ for $x \in N$, $f(1) = f(2016)$ and if the sum of digit of number $[f(1) + f(2) + \ldots + f(2015)]$ is $4k$, then the value of k is

8. If $\sin\theta + \cos\theta = 1$, then find the minimum value of $(1 + \text{cosec}\,\theta)(1 + \sec\theta)$.

Section II

One or More than One Options Correct Type

9. If z is a unimodular complex number such that $\text{Re}(z-1) + \text{Re}(z^2) = \int_0^{\pi/2} \sin x \log|\sin x - \cos x|\,dx$, then

(a) $z + \bar{z} = -2$ (b) $z + \bar{z} = 1$

(c) $\arg(z) = \frac{\pi}{3}$ (d) $\arg(z) = \pi$

10. If $a = \tan(\alpha+\beta) - \tan(\alpha-\beta)$, $b = \tan(\alpha+\beta)\tan(\alpha-\beta)$, then

(a) $a = \frac{2\sin 2\beta}{\cos 2\alpha + \cos 2\beta}$ (b) $\tan\left(\frac{\pi}{4} + 2\beta\right) = \frac{1+a+b}{1-a+b}$

(c) $a = \frac{\sin 2\alpha}{\cos^2\alpha - \sin^2\beta}$ (d) $b = \frac{\sin^2\alpha + \cos^2\beta - 1}{\cos^2\alpha + \cos^2\beta - 1}$

11. If a, b, c are three positive numbers in AP and $p = \frac{a+8b}{2b-a} + \frac{8b+c}{2b-c}$, then the value of p^2 can be

(a) 256 (b) 225 (c) 361 (d) 324

12. If $(a\sec\theta, b\tan\theta)$ and $(a\sec\phi, b\tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\theta}{2}\tan\frac{\phi}{2}$ is equal to

(a) $\frac{e-1}{e+1}$ (b) $\frac{1-e}{1+e}$ (c) $\frac{1+e}{1-e}$ (d) $\frac{e+1}{e-1}$

13. Three players A, B, C toss a coin cyclically in that order that is $ABCABCAB\ldots$ till a head is got. Let p be the probability that a coin shows a head. Let α, β, γ be, respectively the probabilities that A, B, C get the first head, then

(a) $\beta = (1-p)\alpha$ (b) $\gamma = \beta(1-p)$

(c) $\alpha = \frac{p(1-p)}{1-(1-p)^3}$ (d) $\gamma = \frac{(1-p)^2 p}{1-(1-p)^3}$

14. Let $f(x) = \int_0^x e^t \sin(x-t)\,dt$, then

(a) $f''\left(\frac{\pi}{2}\right) - f'\left(\frac{\pi}{2}\right) = -1$ (b) $f''(0) - f'(0) = 1$

(c) $(f''(x) - f(x))_{\max} = \sqrt{2}$ (d) $f''\left(\frac{\pi}{2}\right) - f\left(\frac{\pi}{2}\right) = 1$

15. If V_1, V_2, V_3 are the volumes of parallelopiped, triangular prism and tetrahedron, respectively. The three coterminous edges of all three figures are the vectors $\hat{i} - \hat{j} - 6\hat{k}$, $\hat{i} - \hat{j} + 4\hat{k}$ and $2\hat{i} - 5\hat{j} + 3\hat{k}$, then

(a) $2V_1 + 3V_3 = 5V_2$ (b) $V_1 + V_2 + V_3 = 60$

(c) $V_1 : V_2 : V_3 = 6 : 3 : 1$ (d) $V_1 + 3V_3 = 3V_2$

16. If $\beta, \gamma \in (0, \pi)$ such that

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+\beta+\gamma) = 0$

and $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+\beta+\gamma) = 0$,

where $f(x) = \sin 2x / (1 + \cos 2x)$ and

$g(x) = \frac{1+\sin x - \cos x}{1+\sin x + \cos x}$, then (here, $f'(x)$ denotes derivative of f with respect to x)

(a) $f'(\beta) = 4$ (b) $f'(\beta) = -2$

(c) $\lim_{x\to\gamma} g(x) = \sqrt{3}$ (d) $\lim_{x\to\gamma} g(x) = \sqrt{2}$

Section III

Linked Comprehension Type

Passage I

Directions (Q. Nos. 17-18) Let $f(x) = x^4 - 6x^3 + 26x^2 - 46x + 65$. All the roots of $f(x) = 0$ are of the form $a_k + ib_k$ for $k = 1, 2, 3, 4$ where, $i = \sqrt{-1}$. Further a_k and b_k are all integers, also $\lambda = |b_1| + |b_2| + |b_3| + |b_4|$ and $\mu = a_1 + a_2 + a_3 + a_4$.

17. If set S is formed whose elements are all $a_i's$ and $b_i's$, then

(a) $\lambda = 5$ (b) $\mu = 6$

(c) $\lambda + \mu = 16$ (d) None of these

18. Roots of the equation are

(a) $1 \pm 2i$ (b) $2 \pm 2i$

(c) $2 \pm 3i$ (d) $-1 \pm 3i$

Passage II

Directions (Q. Nos. 19-20) Integrals of the form $\int \frac{x^2+1}{x^4+kx^2+1}\,dx$, $\int \frac{x^2-1}{x^4+kx^2+1}\,dx$, where, $k \in R$, can be integrated by the following method

(i) Divide the numerator and denominator by x^2

(ii) Put $x + \frac{1}{x} = t$ or $x - \frac{1}{x} = t$ as required.

19. If $\int \frac{dx}{x^4+1} = \frac{1}{2\sqrt{2}} \tan^{-1} f(x) - \frac{1}{4\sqrt{2}} \log g(x) + C$, then

(a) $f(x) = \frac{x^2-1}{4\sqrt{2}x}$

(b) $g(x) = \frac{x^2-1}{4\sqrt{2}x}$

(c) $g(x) = \frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}$

(d) $f(x) = \frac{x^2-1}{\sqrt{2}x}$

20. If $\int (\sqrt{\tan x} + \sqrt{\cot x})\,dx = A \tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{B}\right) + C$, then

(a) $A^2 + B^2 = 4$ (b) $A = \sqrt{2}$

(c) $B = \sqrt{3}$ (d) $A = 1$

Answers

Paper 1

1. (5) **2.** (2) **3.** (4) **4.** (3) **5.** (7) **6.** (0) **7.** (5) **8.** (2) **9.** (a,b,c,d) **10.** (c,d)
11. (a,b,d) **12.** (a,c) **13.** (a,d) **14.** (b,d) **15.** (a,b,c) **16.** (a,d) **17.** (a,b,d) **18.** (b,c)

19. A → q, B → r, C → s, D → p **20.** A → q, B → s, C → p, D → r

Paper 2

1. (5) **2.** (0) **3.** (8) **4.** (9) **5.** (2) **6.** (2) **7.** (3) **8.** (9) **9.** (a,b,c,d) **10.** (a,b,d)
11. (c,d) **12.** (b,c) **13.** (a,b,d) **14.** (b,c,d) **15.** (a,c,d) **16.** (a,c) **17.** (b,c) **18.** (a,c) **19.** (c,d) **20.** (a,b)

MATHEMATICS IN action!

300 Year-old-mystery

We all know about Pythagoras' theorem. "If a, b and c are the sides of a right-angled triangle, with c the side opposite to right angle, then $a^2 + b^2 = c^2$" and there are infinitely many triplet a, b and c of natural numbers which satisfy this equation, e.g. (3, 4, 5), (5, 12, 13) etc. But the question was that, can we find triplet of natural numbers that satisfy $a^3 + b^3 = c^3$ or $a^4 + b^4 = c^4$ or $a^5 + b^5 = c^5$. More generally, can we find triples of natural numbers that satisfy $a^n + b^n = c^n$, where n is any natural number.

Then, a 17th century mathematician convinced himself that such triplet don't exist for any natural number $n > 2$ and state that, "There are no whole number solutions to the equation $x^n + y^n + z^n$ when n is greater than 2."

Over the next three centuries, generations of mathematicians tried and failed to find the proof of that theorem. Finally, a professor proved this theorem and for that he has been awarded the 2016 Abel prize.

SHORTCUT Methods

Here, we are introducing some important shortcut methods on sequence and series to save your precious time in tedious calculations based problem asked in various engineering entrance exams. Use these shortcuts to practice questions and be acquainted with it.

1. In an AP

(i) $(2r+1)$ numbers, $r \in N$, can be taken as $a-rd$, $a-(r-1)d, \ldots, a-d, a, a+d, \ldots, a+(r-1)d, a+rd$

(ii) $(2r)$ numbers, $r \in N$, can be taken as $a-(2r-1)d$, $a-(2r-3)d, \ldots, a-3d, a-d, a+d, a+3d, \ldots, a+(2r-3)d, a+(2r-1)d$.

Example *If the roots of the equation $x^3-12x^2+39x-28=0$ are in AP, then their common difference will be*

(a) ± 1 (b) ± 2

(c) ± 3 (d) ± 4

Sol. ***(c)*** Let the roots be $a-d$, a, $a+d$. Then

$(a-d)+a+(a+d)=12 \Rightarrow a=4$

and $(a-d)\,a\,(a+d)=28 \Rightarrow 4(4^2-d^2)=28$

$\Rightarrow \quad 16-d^2=7$

$\Rightarrow \quad d^2=9$

$\Rightarrow \quad d=\pm 3$

2. If $T_n=pn+q$, i.e. a linear expression in n, then the sequence will form an AP with common difference p and first term $p+q$.

Example *If the nth term of a sequence is $(2n-1)$, then the sum of its first n terms will be*

(a) n^2-1 (b) $(2n-1)^2$

(c) n^2 (d) n^2+1

Sol. ***(c)*** Clearly, given sequence is an AP, with first term $2-1=1$ and common difference $=2$.

Now, $S_n=\frac{n}{2}[a+l]=\frac{n}{2}[1+2n-1]$

[$\because$ last term will be nth term]

$=\frac{n}{2}\times 2n=n^2$

3. If $S_n=an^2+bn+c$, i.e. a quadratic function in n, then the series is in AP with common difference $d=2a=$ twice the coefficient of n^2.

Example *If $S_n=nP+\frac{1}{2}n(n-1)Q$, where S_n denotes the sum of the first n terms of an AP, then the common difference is*

(a) $P+Q$ (b) $2P+3Q$

(c) $2Q$ (d) Q

Sol. ***(d)*** We have, $S_n=nP+\frac{1}{2}n(n-1)Q$

$=nP+\frac{n^2}{2}Q-\frac{nQ}{2}$

Clearly, common difference $=2\cdot\left(\frac{Q}{2}\right)=Q$

4. If $a_1, a_2, a_3, \ldots, a_n$ are in AP, where $a_i>0$ for all i, then

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{(n-1)}{\sqrt{a_1}+\sqrt{a_n}}$$

Example $\frac{1}{\sqrt{2}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{11}}+\ldots$ *to n terms, is equal to*

(a) $\frac{\sqrt{3n+2}-\sqrt{2}}{3}$ (b) $\frac{(n-1)}{\sqrt{3n-1}+\sqrt{2}}$

(c) $\frac{1}{\sqrt{2+3n}+\sqrt{2}}$ (d) None of these

Sol. ***(a)*** Clearly, arithmetic progression corresponding to given series is

2, 5, 8, ..., whose nth term is given by

$a_n=a+(n-1)d=2+(n-1)3=3n-1$

Now, the sum of n terms of given series is given by

$$\frac{n}{\sqrt{a_1}+\sqrt{a_{n+1}}}=\frac{n}{\sqrt{2}+\sqrt{3(n+1)-1}}$$

$$=\frac{n}{\sqrt{2}+\sqrt{3n+2}}=\frac{\sqrt{3n+2}-\sqrt{2}}{3}$$

5. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in AP, then

(i) $\dfrac{1}{a_1a_2\ldots a_r}+\dfrac{1}{a_2a_3\ldots a_{r+1}}+\ldots+\dfrac{1}{a_na_{n+1}\ldots a_{n+r-1}}$

$$=\frac{1}{(r-1)(a_2-a_1)}\left[\frac{1}{a_1a_2\ldots a_{r-1}}-\frac{1}{a_{n+1}a_{n+2}\ldots a_{n+r-1}}\right]$$

(ii) $a_1a_2\ldots a_r+a_2a_3\ldots a_{r+1}+\ldots+a_na_{n+1}\ldots a_{n+r-1}$

$=\dfrac{1}{(r+1)(a_2-a_1)}[a_na_{n+1}\ldots a_{n+r}-a_0a_1a_2\ldots a_r]$,

where $a_0=a_1-d$

Example *Sum to n terms of the series* $\dfrac{1}{(1+x)(1+2x)}+\dfrac{1}{(1+2x)(1+3x)}+\dfrac{1}{(1+3x)(1+4x)}+\ldots$ *is*

(a) $\dfrac{nx}{(1+x)(1+nx)}$ (b) $\dfrac{n}{(1+x)[1+(n+1)x]}$

(c) $\dfrac{x}{(1+x)[1+(n-1)x]}$ (d) None of these

Sol. ***(b)*** As $(1+x), (1+2x), (1+3x)\ldots$ are in AP, therefore the sum to n terms of the given series

$$=\frac{1}{(2-1)(x)}\left[\frac{1}{1+x}-\frac{1}{(1+x)+(n+1-1)x}\right]$$

$$=\frac{n}{(1+x)[1+(n+1)x]}$$

6. In a GP,

(i) $(2m+1)$ numbers, $m\in N$, can be taken as $\dfrac{a}{r^m}, \dfrac{a}{r^{m-1}}, \ldots, \dfrac{a}{r}, a, ar, \ldots, ar^{m-1}, ar^m$

(ii) $(2m)$ numbers, $m\in N$, can be taken as

$$\frac{a}{r^{2m-1}}, \frac{a}{r^{2m-3}}, \ldots, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, \ldots, ar^{2m-3}, ar^{2m-1}$$

Example *If the product of three consecutive terms of GP is* 216 *and the sum of product of pairwise is* 156, *then the numbers will be*

(a) 1, 3, 9 (b) 2, 6, 18 (c) 3, 9, 27 (d) 2, 4, 8

Sol. ***(b)*** Let the numbers be $\dfrac{a}{r}, a, ar$. Then, we have $\dfrac{a}{r}\cdot a\cdot ar=216$

$\Rightarrow \quad a^3=216 \Rightarrow a^3=6^3 \Rightarrow a=6$

Also, we have $\dfrac{a}{r}\cdot a+a\cdot ar+ar\cdot\dfrac{a}{r}=156$

$\Rightarrow \quad \dfrac{1}{r}+r+1=\dfrac{13}{3} \Rightarrow 3(r^2+r+1)=13r$

$\Rightarrow 3r^2+3r-13r+3=0 \Rightarrow 3r^2-10r+3=0 \Rightarrow r=\dfrac{1}{3}$ or 3

Hence, the numbers are 2, 6, 18.

7. If $a_1, a_2, a_3, \ldots, a_{n-1}, a_n$ are in GP, then $a_1a_n=a_2a_{n-1}=a_3a_{n-2}=\ldots$ and if $a_1, a_2, a_3\ldots a_n$ are ini AP then $(a_2+a_{n-1})=(a_3+a_{n-2})=\ldots=a_1+a_n$

Example *If* $a, a_1, a_2, \ldots, a_{2n}, b$ *are in arithmetic progression and* $a, g_1, g_2, \ldots g_{2n}, b$ *are in geometric progression and h is the harmonic mean of a and b, then* $\dfrac{a_1+a_{2n}}{g_1g_{2n}}+\dfrac{a_2+a_{2n-1}}{g_2g_{2n-1}}+\ldots+\dfrac{a_n+a_{n+1}}{g_ng_{n+1}}$ *is equal to*

(a) $2nh$ (b) n/h (c) nh (d) $2n/h$

Sol. ***(d)*** Since $a, a_1, a_2, \ldots, a_{2n}, b$ are in AP.

$\therefore a_1+a_{2n}=a_2+a_{2n-1}=\ldots=a_n+a_{n+1}=a+b$ ...(i)

Also, $a, g_1, g_2, \ldots, g_{2n}, b$ are in GP.

$\therefore \quad g_1\cdot g_{2n}=g_2\cdot g_{2n-1}=\ldots=g_n\,g_{n+1}=ab$ ...(ii)

Now, from Eqs. (i) and (ii), we get

$$\frac{a_1+a_{2n}}{g_1g_{2n}}+\frac{a_2+a_{2n-1}}{g_2\cdot g_{2n-1}}+\ldots+\frac{a_n+a_{n+1}}{g_ng_{n+1}}$$

$$=\frac{a+b}{ab}+\frac{a+b}{ab}+\ldots+\frac{a+b}{ab}$$

$$=n\cdot\frac{(a+b)}{(ab)}=\frac{2n}{h}$$

8. If l is the last term of the GP, then $S_n=\dfrac{lr-a}{r-1}, r\neq 1$

Example *If the sum of n terms of a GP is* 255, *nth term is* 128 *and common ratio is* 2, *then first term will be*

(a) 1 (b) 3 (c) 7 (d) None of these

Sol. ***(a)*** Here, $S_n=255, l=128$ and $r=2$

$\because \quad S_n=\dfrac{lr-a}{r-1}$

$\therefore \quad 255=\dfrac{128\times 2-a}{2-1}$

$\Rightarrow \quad 255=256-a$

$\Rightarrow \quad a=256-255$

$\Rightarrow \quad a=1$

9. If T_p and T_q of any AP are given, then formula for obtaining T_n is $\dfrac{T_n-T_p}{n-p}=\dfrac{T_q-T_p}{q-p}$

Example *If the pth term of an AP is q and qth term is p, then its rth term will be*

(a) $p+q+r$ (b) $p+q-r$ (c) $p+r-q$ (d) $p-q-r$

Sol. ***(b)*** The nth term of given AP is given by

$$\frac{T_n-q}{n-p}=\frac{p-q}{q-p}=-1 \qquad (\because T_p=q \text{ and } T_q=p)$$

$\Rightarrow \quad T_n-q=p-n$

$\Rightarrow \quad T_n=q+p-n$

Thus, $\quad T_r=q+p-r$

10. If T_p and T_q of any GP are given, then formula for obtaining T_n is $\left(\dfrac{T_n}{T_p}\right)^{\frac{1}{n-p}}=\left(\dfrac{T_q}{T_p}\right)^{\frac{1}{q-p}}$

Example *If $(p+q)$th term of a GP is m and $(p-q)$th term is n, then the pth term will be*

(a) $\frac{m}{n}$ (b) $\sqrt{mn}$ (c) mn (d) 0

Sol. **(b)** We have, $T_{p+q} = m$ and $T_{p-q} = n$

Now, the general term say T_r is given by

$$\left(\frac{T_r}{m}\right)^{\frac{1}{r-(p+q)}} = \left(\frac{n}{m}\right)^{\frac{1}{(p-q)-(p+q)}}$$

$$\Rightarrow \quad \left(\frac{T_r}{m}\right)^{\frac{1}{r-(p+q)}} = \left(\frac{n}{m}\right)^{-\frac{1}{2q}}$$

Now, on putting $r = p$, we get

$$\left(\frac{T_p}{m}\right)^{\frac{1}{-q}} = \left(\frac{n}{m}\right)^{-\frac{1}{2q}} \Rightarrow \left(\left(\frac{T_p}{m}\right)^2\right)^{-\frac{1}{2q}} = \left(\frac{n}{m}\right)^{-\frac{1}{2q}}$$

$$\Rightarrow \quad \frac{T_p^2}{m^2} = \frac{n}{m} \Rightarrow T_p^2 = \frac{n}{m} \times m^2 \Rightarrow T_p = \sqrt{nm}$$

11. If $a_1, a_2, a_3, \ldots, a_n$ $(a_i > 0, \forall\, i)$ are in GP, then $\log a_1, \log a_2, \log a_3, \ldots, \log a_n$ are in AP and *vice-versa*. Thus, if $a^{x_1}, a^{x_2}, a^{x_3} \ldots, a^{x_n}$ $(a > 0$ and $a \neq 1)$ are in GP, then $x_1, x_2, \ldots, x_n$ are in AP.

Example *If $\log 2, \log(2^x - 1), \log(2^x + 3)$ are in AP, then the value of x is*

(a) $\frac{5}{2}$ (b) $\log_3 5$

(c) $\log_2 5$ (d) $\log_5 3$

Sol. **(c)** As $\log 2, \log(2^x - 1), \log(2^x + 3)$ are in AP, therefore $2, 2^x - 1, 2^x + 3$ are in GP.

Thus, we have $(2^x - 1)^2 = 2 \cdot (2^x + 3)$

$\Rightarrow \quad (2^x)^2 + (1)^2 - 2 \cdot 2^x = 2 \cdot 2^x + 6$

$\Rightarrow \quad (2^x)^2 - 4 \cdot 2^x - 5 = 0$

$\Rightarrow \quad (2^x + 1)(2^x - 5) = 0$

$\Rightarrow \quad 2^x - 5 = 0 \qquad [\because 2^x + 1 \neq 0]$

$\Rightarrow \quad 2^x = 5$

$\Rightarrow \quad x = \log_2 5$

12. If pth, qth and rth terms of a AP/GP/HP are in AP/GP/HP, then p, q, r are in AP and conversely if p, q, r are in AP then the pth, q th and rth terms of AP/GP/HP are in AP/GP/HP, respectively.

Example *If $64, 16, 4$ are the Pth, Qth and Rth terms of a GP, then $P + R$ is equal to*

(a) Q (b) $2Q$

(c) $3Q$ (d) $4Q$

Sol. **(b)** Since, Pth, Qth and Rth terms of a GP are in GP, therefore P, Q, R are in AP. Hence, $2Q = P + R$

13. If the pth, qth and rth terms of an AP are in GP, then common ratio of the GP is $\frac{q-r}{p-q}$.

Example *If 3rd, 7th and 12th terms of an AP are three consecutive terms of a GP, then the common ratio of the GP is*

(a) $\frac{5}{4}$ (b) $\frac{9}{4}$ (c) $\frac{2}{9}$ (d) $\frac{1}{2}$

Sol. **(a)** Clearly, common ratio $r = \frac{7-12}{3-7} = \frac{-5}{-4} = \frac{5}{4}$

14. $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ will be

(i) AM between a and b, if $n = 0$

(ii) GM between a and b, if $n = -\frac{1}{2}$

(iii) HM between a and b, if $n = -1$

Example *The value of n for which $\frac{x^{n+1} + y^{n+1}}{x^n + y^n}$ is the geometric mean of x and y is*

(a) $n = -\frac{1}{2}$ (b) $n = \frac{1}{2}$ (c) $n = 1$ (d) $n = -1$

Sol. **(a)** Clearly, for $n = -\frac{1}{2}$, $\frac{x^{n+1} + y^{n+1}}{x^n + y^n}$ is GM of x and y.

15. If a and b are two positive real numbers and A, G, H are arithmetic mean, geometric mean and harmonic mean between a and b, then

(i) $A \geq G \geq H$ and equality hold only when $a = b$

(ii) A, G, H are in GP, i.e. $G^2 = AH$

(iii) The numbers a and b are given by $A \pm \sqrt{A^2 - G^2}$.

(iv) Equation having a and b as its roots is $x^2 - 2Ax + G^2 = 0$

Example *If a, b, c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is*

(a) less than 1 (b) equal to 1

(c) greater than 1 (d) any real number

Sol. **(a)** Since, a and b are unequal.

$$\therefore \quad \frac{a^2 + b^2}{2} > \sqrt{a^2 b^2} \quad [\because \text{AM} > \text{GM for unequal numbers}]$$

$$\Rightarrow \quad a^2 + b^2 > 2ab$$

Similarly, $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$

Hence, $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$

$$\Rightarrow \quad ab + bc + ca < 1$$

Example *The HM of two numbers is 4. If their AM is A and GM is G satisfy the relation $2A + G^2 = 27$, then the numbers are*

(a) 1, 3 (b) 2, 6 (c) 3, 6 (d) 6, 2

Sol. **(c)** We know that, $G^2 = AH$

$$\therefore \quad G^2 = 4A \qquad (\because H = 4) \ldots (i)$$

$$\because \quad 2A + G^2 = 27 \Rightarrow 2A + 4A = 27$$

$$\Rightarrow \quad 6A = 27 \quad \Rightarrow \quad A = \frac{9}{2}$$

Now, from Eq. (i), we get $G^2 = 4 \cdot \frac{9}{2} = 18$

Hence, the numbers are given by $A \pm \sqrt{A^2 - G^2}$

$$= \frac{9}{2} \pm \sqrt{\frac{81}{4} - 18} = \frac{9}{2} \pm \sqrt{\frac{81 - 72}{4}}$$

$$= \frac{9}{2} \pm \sqrt{\frac{9}{4}} = \frac{9}{2} \pm \frac{3}{2} = 6 \text{ and } 3$$

16. If A, G and H are respectively the arithmetic mean, geometric mean and harmonic mean of three numbers a, b and c, then the equation having a, b and c as its roots is $x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$

Example *The AM, GM and HM of the roots of the equation, $8x^3 - 14x^2 + 7x - 1 = 0$, are respectively*

(a) $\frac{3}{7}, \frac{7}{12}, \frac{1}{2}$ (b) $\frac{7}{12}, \frac{1}{2}, \frac{3}{7}$ (c) $\frac{7}{12}, \frac{3}{7}, \frac{1}{2}$ (d) None of these

Sol. **(b)** We have, $8x^3 - 14x^2 + 7x - 1 = 0$

$$\Rightarrow \quad x^3 - \frac{14}{8}x^2 + \frac{7}{8}x - \frac{1}{8} = 0$$

On comparing this with $x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$, we get

$$3A = \frac{14}{8}, \frac{3G^3}{H} = \frac{7}{8} \text{ and } G^3 = \frac{1}{8}$$

$$\Rightarrow \quad A = \frac{14}{24}, \frac{3}{H} \cdot \frac{1}{8} = \frac{7}{8} \text{ and } G = \frac{1}{2}$$

$$\Rightarrow \quad A = \frac{7}{12}, H = \frac{3}{7} \text{ and } G = \frac{1}{2}$$

17. If $A_1, A_2, \ldots, A_n$ are n AM's between a and b, then

$A_1 + A_2 + \ldots + A_n = n$ (AM of a and b)

Thus, $\frac{\text{sum of } m \text{ AM's}}{\text{sum of } n \text{ AM's}} = \frac{m}{n}$

Example *Between two numbers, whose sum is $2\frac{1}{6}$, an even number of arithmetic means are inserted. If the sum of these means exceeds their number by unity, then the number of means are*

(a) 12 (b) 10 (c) 8 (d) 6

Sol. **(a)** Let $2n$ arithmetic means $A_1, A_2, A_3, \ldots, A_{2n}$ be inserted between a and b. Then,

$$A_1 + A_2 + \ldots + A_{2n}$$

$$= 2n\left(\frac{a+b}{2}\right) = \left(\frac{\frac{13}{6}}{2}\right) \times 2n = \frac{13n}{6}$$

Also, it is given that,

$$A_1 + A_2 + \ldots + A_{2n} = 2n + 1$$

$$\Rightarrow \quad \frac{13n}{6} = 2n + 1 \Rightarrow 13n = 12n + 6$$

$$\Rightarrow \quad n = 6$$

18. If $G_1, G_2, \ldots G_n$ are n GM's between a and b, then

$G_1 \cdot G_2 \ldots G_n = (\text{GM of } a \text{ and } b)^n$ and $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Thus, the product of n geometric means between a and $\frac{1}{a}$ is 1.

Example *The product of three geometric means between 4 and $\frac{1}{4}$ will be*

(a) 4 (b) 2 (c) – 1 (d) 1

Sol. **(d)** Clearly, the product of three geometric means between 4 and $\frac{1}{4}$ is 1.

19. If $H_1, H_2, \ldots, H_n$ are n HM's between a and b, then

$$\frac{1}{H_1} + \frac{1}{H_2} + \ldots + \frac{1}{H_n} = n\left(\frac{1}{\text{HM of } a \text{ and } b}\right)$$

Example *If H_1, H_2 are two harmonic means between two positive numbers a and b $(a \neq b)$, A and G are the arithmetic and geometric means between a and b, then $\frac{H_2 + H_1}{H_2 H_1}$ is equal to*

(a) $\frac{A}{G}$ (b) $\frac{2A}{G}$ (c) $\frac{2A}{G^2}$ (d) $\frac{A}{G^2}$

Sol. **(c)** Consider, $\frac{H_2 + H_1}{H_2 H_1} = \frac{1}{H_1} + \frac{1}{H_2}$

$$= 2\left(\frac{1}{\text{HM of } a \text{ and } b}\right) = 2\left\{\frac{1}{\left(\frac{2ab}{a+b}\right)}\right\}$$

$$= \frac{2(a+b)}{2ab} = \frac{a+b}{ab} = \frac{2A}{G^2} \qquad \left(\because A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}\right)$$

20. If A_1, A_2 are the AM's, G_1, G_2 are the GM's and H_1, H_2 are the HM's between any two numbers, then

$$\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2}$$

Example *If A_1, A_2 are the two AM's between two numbers a and b and G_1, G_2 are two GM's between same two numbers, then $\frac{A_1 + A_2}{G_1 G_2}$ is equal to*

(a) $\frac{a+b}{ab}$ (b) $\frac{a+b}{2ab}$ (c) $\frac{2ab}{a+b}$ (d) $\frac{ab}{a+b}$

Sol. **(a)** Clearly, $\frac{A_1 + A_2}{G_1 G_2} = \frac{H_1 + H_2}{H_1 H_2} = \frac{1}{H_1} + \frac{1}{H_2}$

$$= \frac{a+2b}{3ab} + \frac{2a+b}{3ab} \qquad \left(\because H_1 = \frac{3ab}{a+2b} \text{ and } H_2 = \frac{3ab}{2a+b}\right)$$

$$= \frac{1}{3ab}(3a+3b) = \frac{a+b}{ab}$$

21. Middle Term of AP/GP/HP

(i) If the number of terms of any AP/GP/HP is odd, then its middle term is AM/GM/HM of first and last terms.

(ii) If the number of terms of any AP/GP/HP is even, then AM/GM/HM of middle two terms is AM/GM/HM of first and last terms.

Example *An AP, a GP and a HP have the same first and last terms and the same odd number of terms. The middle terms of the three series are in*

(a) AP *(b)* GP *(c)* HP *(d)* None of these

Sol. ***(b)*** Let a and b be the first and last terms of the three progressions, each having $(2n+1)$ terms. Then,

middle term of AP = AM of a and $b = \frac{a+b}{2}$

middle term of GP = GM of a and $b = \sqrt{ab}$

and middle term of HP = HM of a and $b = \frac{2ab}{a+b}$

Now, as we know $GM^2 = AM \times HM$ for two numbers, therefore middle terms are in GP.

22. If number of terms of an AP is odd, then the sum of terms is equal to product of middle term and number of terms.

Example *If* $\sum_{j=1}^{21} a_j = 693$, *where* $a_1, a_2, \ldots, a_{21}$ *are in AP, then* $\sum_{i=0}^{10} a_{2i+1}$ *is equal to*

(a) 361 *(b)* 396 *(c)* 363 *(d)* data insufficient

Sol. ***(c)*** Clearly, $\sum_{j=1}^{21} a_j = 693 = 21$ (middle term)

$$= 21 \cdot a_{11}$$

$$= 21\,\frac{(a_1 + a_{21})}{2} \qquad [\because \text{middle term = AM of first and last term}]$$

$\therefore \quad a_{11} = 33$ and $a_1 + a_{21} = 66$

Since, $a_2 + a_{20} = a_3 + a_{19} = \ldots = a_9 + a_{13} = a_{10} + a_{12} = a_1 + a_{21}$

$\therefore \quad \sum_{i=0}^{10} a_{2i+1} = 5 \times (a_1 + a_{21}) + a_{11}$

$$= 5 \times 66 + 33 = 363$$

23. Product of a GP

(i) If number of terms is odd, then product of terms = $(\text{middle term})^{\text{number of terms}}$

(ii) If number of terms is even, then product of terms = $(\text{GM of middle terms})^{\text{number of terms}}$

(iii) If a and l are the first and last terms respectively of a GP, then product of all terms = $(al)^{\frac{n}{2}}$, where n is the number of terms.

Example *If fourth term of a GP is 3, then the product of the first seven terms is*

(a) 3^4 *(b)* 3^7 *(c)* 7^4 *(d)* 4^7

Sol. ***(b)*** As number of terms are odd, therefore product of terms

$= (\text{middle term})^7 = (a_4)^7 = 3^7$

24. Value of Recurring Decimal

(i) If $R = 0.\overline{Y}$, then $R = \frac{Y}{10^y - 1}$

(ii) If $R = 0.X\overline{Y}$, then $R = \frac{XY - X}{10^{x+y} - 10^x}$, where x denotes the number of digits in X and y denotes the number of digits in Y.

Example $2.\overline{357} =$

(a) $\frac{2335}{1001}$ *(b)* $\frac{2370}{997}$ *(c)* $\frac{2355}{999}$ *(d)* None of these

Sol. ***(c)*** Clearly, $2.\overline{357} = 2 + 0.\overline{357}$

$$= 2 + \frac{357}{10^3 - 1} = 2 + \frac{357}{999} = \frac{2355}{999}$$

25. Inequalities Involving Arithmetic Mean of mth power

Let $a_1, a_2, a_3, \ldots, a_n$ be n positive real numbers (not all equal) and m be a real number, then

$$\frac{a_1^m + a_2^m + \ldots + a_n^m}{n} > \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^m, \forall\, m \in R - [0, 1]$$

$$< \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^m, \forall\, m \in (0, 1)$$

$$= \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^m, \forall\, m \in \{0, 1\}$$

Example *If* $a, b > 0$ *such that* $a^3 + b^3 = 2$, *then* $(a+b)$ *is*

(a) ≥ 2 *(b)* > 2 *(c)* ≤ 2 *(d)* < 2

Sol. ***(c)*** If $a \neq b$, then we have $\frac{a^3 + b^3}{2} > \left(\frac{a+b}{2}\right)^3$

$$\Rightarrow \quad \frac{2}{2} > \left(\frac{a+b}{2}\right)^3 \qquad [\because a^3 + b^3 = 2]$$

$$\Rightarrow \quad \left(\frac{a+b}{2}\right)^3 < 1$$

$$\Rightarrow \quad 0 < \frac{a+b}{2} < 1 \qquad [\because a + b > 0]$$

$$\Rightarrow \quad a + b < 2$$

and if $a = b$, then $a^3 + a^3 = 2$

$$\Rightarrow \quad 2a^3 = 2 \Rightarrow a^3 = 1$$

$$\Rightarrow \quad a = 1 \qquad [\because a \in R^+]$$

$$\Rightarrow \quad a + b = a + a = 2$$

Hence, $a + b \leq 2$.

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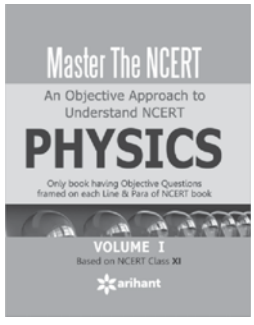
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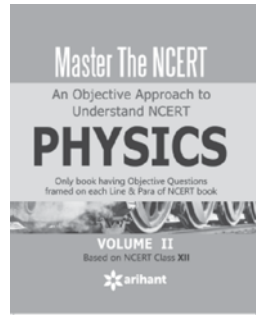
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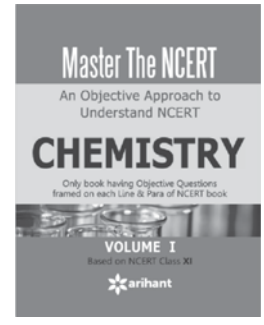
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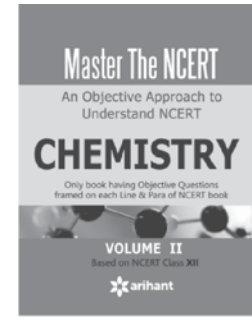
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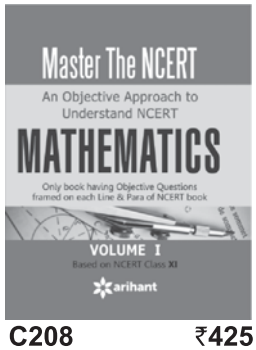
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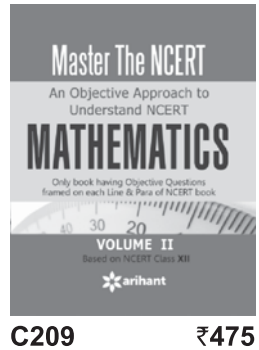
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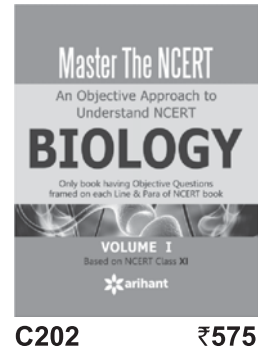
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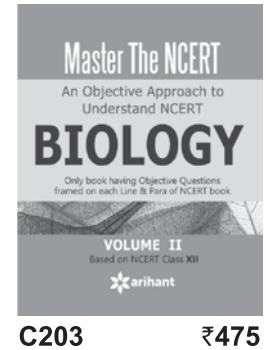
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OLYMPIAD PRACTICE

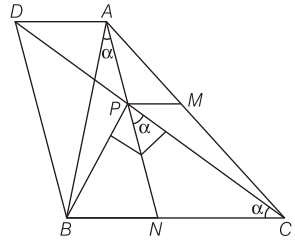
Suggested practice questions for Indian National Maths Olympiad (INMO) and Regional Maths Olympiad (RMO)

1. Let ABC be a triangle and P be an interior point such that $\angle BPC = 90°$, $\angle BAP = \angle BCP$. Let M, N be the mid-points of AC, BC respectively. Suppose $BP = 2PM$. Prove that A, P and N are collinear.

Sol. Extend CP to D such that

$$CP = PD$$

Let $$\angle BCP = \angle BAP = \alpha$$



Observe that, BP is the perpendicular bisector of CD.

Hence, $BC = BD$ and BCD is an isosceles triangle.

Thus, $$\angle BDP = \alpha$$

But $$\angle BDP = \alpha = \angle BAP$$

In turn, we conclude that

$$\angle DAB = \angle DPB = 90°$$

Since, P is the mid-point of CP (by construction) and M is the mid-point of CA (given). It follows that PM is parallel to DA and $DA = 2PM = BP$.

Thus, $DBPA$ is an isosceles trapezium and DB is parallel to PA.

Hence, we get

$$\angle DPA = \angle BAP = \angle BCP = \angle NPC$$

The last equality follows from the fact that, $\angle BPC = 90°$ and N is the mid-point of CB, so that $NP = NC = NB$ for the right angled ΔBPC.

If follows that, A, P and N are collinear.

Alternate Solution

We use coordinate geometry. Let us take $P(0,0)$ and the coordinate axis along PC and PB. We take $C(c,0)$ and $B(0, b)$. Let $A = (u,v)$.

Then, $N = \left(\frac{c}{2}, \frac{b}{2}\right)$ and $M = \left(\frac{u+c}{2}, \frac{v}{2}\right)$

The condition $PB = 2PM$ translates to $(u+c)^2 + v^2 = b^2$.

We observe that, the slope of $CP = 0$ and that of CB is $-\frac{b}{c}$ that of PA is $\frac{v}{u}$ and that of BA is $\frac{(v-b)}{u}$.

Taking proper signs, we can convert $\angle PCB = \angle PAB$ *via*. tan function to the following relation

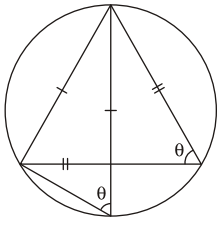
$$u^2 + v^2 - vb = -cu$$

Thus, we obtain $u(u+c) = v(b-v), c(c+u) = b(b-v)$

It follows that, $\frac{v}{u} = \frac{b}{c}$. But then we get that the slope of AP and PN are the same. We conclude that, A, P and N are collinear.

2. T_1 is an isosceles triangle with circumcircle K. Let T_2 be another isosceles triangle inscribed in K whose base is one of the equal side of T_1 and which overlaps the interior of T_1. Similarly, create isosceles triangle from T_2, T_3, T_4 and so on. Do the triangle T_n approaches an equilateral triangle as $n \to \infty$?

Sol. Note that, the base angle of T_n is equal to the angle opposite the base of T_{n+1}(as the figure indicates). Therefore, if θ is base angle for T_n, then the base angle for next triangle T_{n+1} is $\frac{180°-\theta}{2}=90°-\frac{\theta}{2}$



Suppose that, θ is the base angle for T_1. Then, the base angle for T_n is

$$90°-\frac{90°}{2}+\frac{90°}{4}-\frac{90°}{8}+\ldots+(-1)^{n-2}\frac{90°}{2^{n-2}}+(-1)^{n-1}\frac{\theta}{2^{n-1}}.$$

Note that the limit of above as $n \to \infty$ is $\frac{90°}{1+\frac{1}{2}}=60°$ by the formula for the sum of infinite geometric series. Since, each T_n is isosceles, the angle of T_n approaches 60° as $n \to \infty$.

3. Determine all the roots, real or complex of the system of simultaneous equations

$$x+y+z=3$$
$$x^2+y^2+z^2=3$$
$$x^3+y^3+z^3=3$$

Sol. Let x, y and z be the roots of cubic equation

$$t^3-at^2+bt-c=0$$

Then, $a=x+y+z=3$

and $2b=2(xy+yz+zx)$

$$=(x+y+z)^2-(x^2+y^2+z^2)=6$$

This cubic equation is satisfied by x, y and z, if we sum these cubic, we obtain

$$(x^3-ax^2+bx-c)+(y^3-ay^2+by-c)+(z^3-az^2+bz-c)=0$$

$$\Rightarrow \quad x^3+y^3+z^3-a(x^2+y^2+z^2)+b(x+y+z)-3c=0$$

$$3-3a+3b-3c=0$$

Since, $a=b=3 \Rightarrow c=1$

$\therefore$ Cubic equation becomes $(t-1)^3=0$ and so the only solution is $(x, y, z) = (1,1,1)$.

4. Find the product of $101\times10001\times100000001\times\ldots\times(10000\ldots01)$, where the last factor has 2^7-1 zeroes between the ones. Find the number of ones in the product.

Sol. Since, $101\times10001\times100000001\times\ldots\times(10000\ldots01)$

$$=(10^2+1)(10^4+1)(10^8+1)\ldots(10^{2^7}+1)$$

$$=(10^2+1)(10^4+1)(10^8+1)\ldots(10^{128}+1)$$

On multiplying and dividing by 10^2-1, we get

$$=\frac{(10^2-1)(10^2+1)}{(10^2-1)}(10^4+1)(10^8+1)\ldots(10^{2^7}+1)$$

$$\frac{1}{(10^2-1)}(10^8-1)(10^8+1)\ldots(10^{128}+1)$$

$$\ldots \quad \ldots \quad \ldots$$

$$=\frac{10^{2^8}-1}{10^2-1}=\frac{(10^2)^{128}-1}{10^2-1}$$

$$=(10^2)^{127}+(10^2)^{126}+\ldots+1$$

$$=10^{254}+10^{252}+\ldots+10^2+1=10101\ldots101$$

5. When the numbers 19779 and 17997 are divided by a certain three digit number, they leave the same remainder. Find this largest such divisor and the remainder. How many such divisor are there?

Sol. Let the divisor be d and the remainder be r, then by Euclidean algorithm, we find

$$19779=dq_1+r \qquad \ldots(i)$$
$$17997=dq_2+r \qquad \ldots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$1782=d(q_1-q_2)$$

$\therefore$ d is a three digit divisors of 1782.

So, possible values of d are 891, 594, 297, 198.

Hence, largest three digit divisor is 891 and the remainder is 177.

6. A leaf is torn from back of a novel. The sum of remaining pages is 15000. What are the page numbers on the torn leaf?

Sol. Let the number of pages in the novel be n.

Since, sum of the number of pages after a leaf is torn, is 15000. Then, sum of the numbers on all the pages must exceed 15000.

i.e. $\frac{n(n+1)}{2}>15000 \Rightarrow n(n+1)>30000$

$\therefore$ $(n+1)^2>n(n+1)>30000>173^2$

$\Rightarrow$ $(n-2)<174 \Rightarrow n<176$

$\therefore$ $172<n<176$

Hence, n could be one of 173, 174, 175

If $n=173$, then $\frac{n(n+1)}{2}=\frac{173\times174}{2}=15051$

Thus, the sum of the numbers on the torn pages

$=15015-15000=51$ and this should be $x+(x+1)$

$=2x+1=51$

$\therefore$ Page numbers on the torn pages are 26, 25.

If $n=174$, then $\frac{n(n+1)}{2}=15225$

Now, the sum of the numbers on the torn pages

$=15225-15000=225$

$\therefore$ Page numbers on the torn pages are 112, 113 but actually the smaller number on the torn page should be odd and hence though it is theoretically correct but not acceptable in reality.

If $n = 175$, then $\frac{n(n+1)}{2}=15400$ and sum of the numbers on the torn page is 400 which is not possible because sum should be odd. So, the numbers on the torn pages are 25, 26 and the number of pages is 173.

Exam held on 14 March, 2016

{EXAM DIARY}

CBSE EXAMINATION PAPER 2016 SOLVED (DELHI)

General Instructions

- All questions are compulsory.
- Please check that this question paper contains 26 questions.
- Questions 1-6 in Section A are very short answer type questions carrying 1 mark each.
- Questions 7-19 in Section B are long answer I type questions carrying 4 marks each.
- Questions 20-26 in Section C are long answer II type questions carrying 6 marks each.
- Please write down the serial number of the question before attempting it.

Section A

1. If matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, then find values of a and b.

2. Find the position vector of a point which divides the join of points with position vectors $\mathbf{a} - 2\mathbf{b}$ and $2\mathbf{a} + \mathbf{b}$ externally in the ratio 2 : 1.

3. The two vectors $\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ represent the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A.

4. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$.

5. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$.

6. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

Section B

7. Show that the equation of normal at any point t on the curve $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 t$ is

$$4(y\cos^3 t - x\sin^3 t) = 3\sin 4t.$$

8. Find $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta}\, d\theta$.

OR

Evaluate $\int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$.

9. Find $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\, dx$.

10. Evaluate $\int_{-1}^{2} |x^3 - x|\, dx$.

11. Find the particular solution of the differential equation

$(1 - y^2)(1 + \log x)dx + 2xydy = 0$, given that $y = 0$ when $x = 1$.

12. Find the general solution of the following differential equation

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

13. Show that the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanar, if $\mathbf{a} + \mathbf{b}$, $\mathbf{b} + \mathbf{c}$ and $\mathbf{c} + \mathbf{a}$ are coplanar.

14. Find the vector and cartesian equations of the line through the point $(1, 2, -4)$ and perpendicular to the two lines

$$\mathbf{r} = (8\hat{\mathbf{i}} - 19\hat{\mathbf{j}} + 10\hat{\mathbf{k}}) + \lambda(3\hat{\mathbf{i}} - 16\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$$

and $\mathbf{r} = (15\hat{\mathbf{i}} + 29\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + \mu(3\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$.

15. Three persons A, B and C apply for a job of manager in a private company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, then find the probability that it is due to the appointment of C.

OR

A and B throw a pair of dice alternately. A wins the game, if he gets a total of 7 and B wins the game, if he gets a total of 10. If A starts the game, then find the probability that B wins.

16. Prove that

$$\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}.$$

OR

Solve for x

$$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$$

17. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15000 per month, then find their monthly incomes using matrix method. This problem reflects which value ?

18. If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, then find the values of $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ and $t=\frac{\pi}{3}$.

OR

If $y = x^x$, then prove that $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$.

19. Find the values of p and q, for which

$$f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2}\\ p, & \text{if } x=\dfrac{\pi}{2}\\ \dfrac{q(1-\sin x)}{(\pi-2x)^2}, & \text{if } x>\dfrac{\pi}{2}\end{cases}$$

is continuous at $x = \pi/2$.

Section C

20. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also, find maximum volume in terms of volume of the sphere.

OR

Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

21. Using integration find the area of the region

$$\{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x, y \ge 0\}.$$

22. Find the coordinate of the point P where the line through $A(3, -4, -5)$ and $B(2, -3, 1)$ crosses the plane passing through three points $L(2, 2, 1)$, $M(3, 0, 1)$ and $N(4, -1, 0)$. Also, find the ratio in which P divides the line segment AB.

23. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also, find mean and variance of the distribution.

24. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 h and that of second machine is 9 h per day. Each unit of product A requires 3 h on both machines and each unit of product B requires 2 h on first machine and 1 h on second machine. Each unit of product A is sold at ₹ 7 profit and that of B at a profit of ₹ 4. Find the production level per day for maximum profit graphically.

25. Let $f : N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

26. Prove that

$$\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2\\ zx-y^2 & xy-z^2 & yz-x^2\\ xy-z^2 & yz-x^2 & zx-y^2\end{vmatrix}$$

is divisible by $(x + y + z)$, and hence find the quotient.

OR

Using elementary transformations, find the inverse of the matrix $A=\begin{pmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}$ and use it to solve the following system of linear equations

$$8x + 4y + 3z = 19$$
$$2x + y + z = 5$$
$$x + 2y + 2z = 7$$

Answers *with* Explanation

1. We have, $A=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$

Since, A is symmetric matrix, therefore we have

$$A^T = A$$

$$\Rightarrow \begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}^T = \begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0 & 3 & 3a\\ 2b & 1 & 3\\ -2 & 3 & -1\end{bmatrix} = \begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$$

$$\Rightarrow 3a=-2 \text{ and } 2b=3$$

$$\Rightarrow a=\frac{-2}{3} \text{ and } b=\frac{3}{2}$$

2. Let $\mathbf{r}$ be the position vector of a point which divides the join of points with position vectors $\mathbf{a}-2\mathbf{b}$ and $2\mathbf{a}+\mathbf{b}$ externally in the ratio 2 : 1. Then

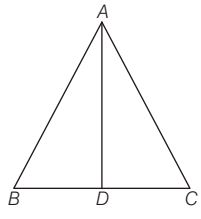
$$\mathbf{r}=\frac{2(2\mathbf{a}+\mathbf{b})-(\mathbf{a}-2\mathbf{b})}{2-1}$$

$$=4\mathbf{a}+2\mathbf{b}-\mathbf{a}+2\mathbf{b}$$

$$=3\mathbf{a}+4\mathbf{b}$$

which is the required position vector.

3. **Given** $\mathbf{AB}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{AC}=3\hat{\mathbf{i}}-\hat{\mathbf{j}}+4\hat{\mathbf{k}}$



To find $|\mathbf{AD}|$, where D is foot of median through A.

Clearly, $\mathbf{AD}=\frac{\mathbf{AB}+\mathbf{AC}}{2}$

$$=\frac{(\hat{\mathbf{j}}+\hat{\mathbf{k}})+(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+4\hat{\mathbf{k}})}{2}$$

$$=\frac{3\hat{\mathbf{i}}+5\hat{\mathbf{k}}}{2}=\frac{3}{2}\hat{\mathbf{i}}+\frac{5}{2}\hat{\mathbf{k}}$$

Hence, $|\mathbf{AD}|=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2}=\frac{1}{2}\sqrt{9+25}=\frac{\sqrt{34}}{2}$

Trick Assume A as origin. Then find position vector of D, which divide the join of B and C in the ratio 1 : 1, using section formula.

4. Let $\mathbf{n}=2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ and $d=5$. Then, vector equation of plane is given by

$$\mathbf{r}\cdot\hat{\mathbf{n}}=d$$

$$\Rightarrow \mathbf{r}\cdot\frac{(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+6\hat{\mathbf{k}})}{\sqrt{2^2+(-3)^2+6^2}}=5$$

$\Rightarrow \mathbf{r}\cdot(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+6\hat{\mathbf{k}})=35$, which is the required equation of the plane.

5. Let $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1 & 1 & 1+\cos\theta\end{vmatrix}$

On applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$, we get

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 0 & \sin\theta & 0\\ 0 & 0 & \cos\theta\end{vmatrix}$$

Now, on expanding along C_1, we get

$$\Delta=1(\sin\theta\cos\theta-0)-0+0$$

$$\Rightarrow \Delta=\sin\theta\cos\theta=\frac{1}{2}(2\sin\theta\cos\theta)=\frac{1}{2}\sin2\theta$$

Clearly, maximum value of Δ is $\frac{1}{2}$ as maximum value of $\sin2\theta$ is 1.

Hence, $\Delta_{\max}=\frac{1}{2}$.

6. **Given** A is a square matrix such that $A^2=I$

To find $(A-I)^3+(A+I)^3-7A$

Consider,

$$(A-I)^3+(A+I)^3-7A$$

$$=A^3-I^3-3A^2I+3AI^2+A^3+I^3+3A^2I+3AI^2-7A$$

$$=2A^3+6AI^2-7A$$

$$=2(A^2\cdot A)+6AI-7A \qquad [\because I^n=I, n\in N]$$

$$=2(I\cdot A)+6AI-7A \qquad [\because A^2=I]$$

$$=2A+6A-7A \qquad [\because I\cdot A=A\cdot I=A]$$

$$=A$$

7. **Given** Equation of curve,

$$x=3\cos t-\cos^3 t \text{ and } y=3\sin t-\sin^3 t$$

To show Equation of normal at any point t on the curve is

$$4(y\cos^3 t-x\sin^3 t)=3\sin 4t$$

Consider, $x=3\cos t-\cos^3 t$

$$\Rightarrow \frac{dx}{dt}=-3\sin t-3\cos^2 t(-\sin t)$$

$$=3\cos^2 t\sin t-3\sin t=3\sin t(\cos^2 t-1)$$

$$=3\sin t(-\sin^2 t) \qquad [\because \sin^2 x+\cos^2 x=1]$$

$$=-3\sin^3 t \qquad \ldots(i)$$

and $y=3\sin t-\sin^3 t$

$$\Rightarrow \frac{dy}{dt}=3\cos t-3\sin^2 t\cos t$$

$$=3\cos t(1-\sin^2 t)=3\cos t(\cos^2 t)$$

$$=3\cos^3 t \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3\cos^3 t}{(-3\sin^3 t)}=-\cot^3(t)$$

Thus, slope of normal at any point t

$$=-\frac{1}{(-\cot^3(t))}=\tan^3(t)$$

Now, equation of normal to the curve is given by

$$(y-(3\sin t-\sin^3 t))=\frac{\sin^3 t}{\cos^3 t}(x-(3\cos t-\cos^3 t))$$

$$\Rightarrow \quad y\cos^3 t-3\sin t\cos^3 t+\sin^3 t\cos^3 t = x\sin^3 t-3\cos t\sin^3 t+\sin^3 t\cos^3 t$$

$$\Rightarrow \quad (y\cos^3 t-x\sin^3 t)=3\sin t\cos^3 t-3\cos t\sin^3 t$$

$$\Rightarrow \quad (y\cos^3 t-x\sin^3 t)=3\sin t\cos t(\cos^2 t-\sin^2 t)$$

$$\Rightarrow \quad (y\cos^3 t-x\sin^3 t)=\frac{3}{2}(2\sin t\cos t)\cos 2t$$

$$\Rightarrow \quad 2(y\cos^3 t-x\sin^3 t)=3\sin 2t\cos 2t$$

$$\Rightarrow \quad 4(y\cos^3 t-x\sin^3 t)=3(2\sin 2t\cos 2t)=3\sin 4t$$

Hence proved.

8. Let $I=\int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta=\int\frac{(3\sin\theta-2)\cos\theta}{5-(1-\sin^2\theta)-4\sin\theta}d\theta$

$$=\int\frac{(3\sin\theta-2)\cos\theta}{\sin^2\theta-4\sin\theta+4}d\theta$$

Now, put $\sin\theta=t$

$$\Rightarrow \quad \cos\theta\,d\theta=dt$$

$$\therefore \quad I=\int\frac{(3t-2)}{t^2-4t+4}dt=\int\frac{(3t-2)}{(t-2)^2}dt$$

Now, let $\quad \frac{3t-2}{(t-2)^2}=\frac{A}{(t-2)}+\frac{B}{(t-2)^2}$

$$\Rightarrow \quad 3t-2=A(t-2)+B$$

On putting $t=2$, we get

$$3(2)-2=B\Rightarrow B=4$$

On comparing the constant term, we get

$$-2=-2A+B$$

$$\Rightarrow \quad -2A=-2-B=-2-4=-6$$

$$\Rightarrow \quad A=3$$

Thus, $\quad I=\int\left\{\frac{3}{(t-2)}+\frac{4}{(t-2)^2}\right\}dt$

$$=3\log|t-2|-\frac{4}{(t-2)}+C$$

$$=3\log|\sin\theta-2|-\frac{4}{(\sin\theta-2)}+C \quad [\because t=\sin\theta]$$

OR

Let $\quad I=\int_0^{\pi}\underset{\text{II}}{e^{2x}}\cdot\underset{\text{I}}{\sin\left(\frac{\pi}{4}+x\right)}dx$

$$=\left[\sin\left(\frac{\pi}{4}+x\right)\cdot\frac{e^{2x}}{2}-\int\cos\left(\frac{\pi}{4}+x\right)\cdot\frac{e^{2x}}{2}dx\right]_0^{\pi}$$

[using integration by parts]

$$=\left[\sin\left(\frac{\pi}{4}+x\right)\cdot\frac{e^{2x}}{2}\right]_0^{\pi}-\frac{1}{2}\int_0^{\pi}e^{2x}\cdot\cos\left(\frac{\pi}{4}+x\right)dx$$

$$=\frac{1}{2}\left[e^{2\pi}\sin\left(\pi+\frac{\pi}{4}\right)-\sin\frac{\pi}{4}\cdot e^0\right]$$

$$-\frac{1}{2}\left[\cos\left(\frac{\pi}{4}+x\right)\cdot\frac{e^{2x}}{2}-\int\left(-\sin\left(\frac{\pi}{4}+x\right)\right)\cdot\frac{e^{2x}}{2}dx\right]_0^{\pi}$$

$$=\frac{1}{2}\left[-e^{2\pi}\sin\frac{\pi}{4}-\sin\frac{\pi}{4}\right]-\frac{1}{4}\left[\cos\left(x+\frac{\pi}{4}\right)\cdot e^{2x}\right]_0^{\pi}$$

$$-\frac{1}{4}\int_0^{\pi}\sin\left(\frac{\pi}{4}+x\right)\cdot e^{2x}dx$$

$$=\frac{-1}{2\sqrt{2}}(e^{2\pi}+1)-\frac{1}{4}\left[\cos\left(\pi+\frac{\pi}{4}\right)\cdot e^{2\pi}-\cos\left(\frac{\pi}{4}\right)\cdot e^0\right]-\frac{1}{4}I$$

$$=\frac{-1}{2\sqrt{2}}(e^{2\pi}+1)-\frac{1}{4}\left[-\cos\left(\frac{\pi}{4}\right)\cdot e^{2\pi}-\cos\left(\frac{\pi}{4}\right)\right]-\frac{1}{4}I$$

$$\Rightarrow \quad \frac{5}{4}I=\frac{-1}{2\sqrt{2}}(e^{2\pi}+1)+\frac{1}{4\sqrt{2}}(e^{2\pi}+1)$$

$$\Rightarrow \quad I=\frac{4}{5}\left[\frac{-1}{2\sqrt{2}}(e^{2\pi}+1)+\frac{1}{4\sqrt{2}}(e^{2\pi}+1)\right]$$

$$=\frac{(e^{2\pi}+1)}{5}\left[-\sqrt{2}+\frac{1}{\sqrt{2}}\right]=\frac{-(e^{2\pi}+1)}{5\sqrt{2}}$$

9. Let $I=\int\frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx=\int\frac{\sqrt{x}}{\sqrt{(a^{\frac{3}{2}})^2-(x^{\frac{3}{2}})^2}}dx$

Put $\quad x^{\frac{3}{2}}=t\Rightarrow\frac{3}{2}x^{\frac{1}{2}}dx=dt$

$$\Rightarrow \quad \sqrt{x}\,dx=\frac{2}{3}dt$$

$$\therefore \quad I=\frac{2}{3}\int\frac{dt}{\sqrt{(a^{\frac{3}{2}})^2-t^2}}$$

$$=\frac{2}{3}\sin^{-1}\left(\frac{t}{a^{\frac{3}{2}}}\right)+C=\frac{2}{3}\sin^{-1}\left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)+C \quad [\because t=x^{3/2}]$$

10. Let $\quad I=\int_{-1}^{2}|x^3-x|dx=\int_{-1}^{2}|x(x^2-1)|dx$

$$=\int_{-1}^{2}|x(x+1)(x-1)|dx$$

Now, consider $x(x+1)(x-1)=0$

$$\Rightarrow \quad x=0 \text{ or } x=-1 \text{ or } x=1$$

So, break the integral at $x=0$ and $x=1$

$$\therefore\ I=\int_{-1}^{0}|x(x+1)(x-1)|dx+\int_0^1|x(x+1)(x-1)|dx+\int_1^2|x(x+1)(x-1)|dx$$

$$=\int_{-1}^{0}x(x^2-1)dx+\int_0^1-(x(x^2-1))dx+\int_1^2x(x^2-1)dx$$

$$=\int_{-1}^{0}(x^3-x)dx-\int_0^1(x^3-x)dx+\int_1^2(x^3-x)dx$$

$$=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}-\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_0^1+\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_1^2$$

$$=-\left(\frac{1}{4}-\frac{1}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)+\left[\left(\frac{2^4}{4}-\frac{2^2}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$=-3\left(\frac{1}{4}-\frac{1}{2}\right)+\left(\frac{16}{4}-2\right)=-3\left(\frac{-1}{4}\right)+2=\frac{11}{4}$$

11. Consider the given differential equation,

$$(1-y^2)(1+\log x)dx+2xydy=0$$

which can be rewritten as

$$(1-y^2)(1+\log x)dx=-2xy\,dy$$

$$\Rightarrow \quad \frac{(1+\log x)dx}{x}=\left(\frac{-2y}{1-y^2}\right)dy$$

On integrating both sides, we get

$$\int\frac{(1+\log x)}{x}dx=\int\frac{(-2y)}{1-y^2}dy$$

$$\Rightarrow \quad \frac{(1+\log x)^2}{2}=\log|1-y^2|+C$$

$$\left[\because \int f(x)\cdot f'(x)dx = \frac{(f(x))^2}{2}+C \text{ and } \int \frac{f'(x)}{f(x)}dx = \log|f(x)|+C\right]$$

Since, it is given that $y = 0$ when $x = 1$, therefore we have

$$\frac{(1+\log 1)^2}{2} = \log|1-0|+C$$

$$\Rightarrow \quad C = \frac{1}{2} \qquad [\because \log 1 = 0]$$

Hence, required particular solution is

$$\frac{(1+\log x)^2}{2} = \log|1-y^2|+\frac{1}{2}$$

$$\Rightarrow \quad (1+\log x)^2 = \log(1-y^2)^2+1$$

12. Consider the given differential equation,

$$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx} = 0. \text{ which can be rewritten as}$$

$$(1+y^2) = -(x-e^{\tan^{-1}y})\frac{dy}{dx}$$

$$\Rightarrow \quad (1+y^2)\frac{dx}{dy} = -x+e^{\tan^{-1}y}$$

$$\Rightarrow \quad (1+y^2)\frac{dx}{dy}+x = e^{\tan^{-1}y}$$

$$\Rightarrow \quad \frac{dx}{dy}+\frac{x}{(1+y^2)} = \frac{e^{\tan^{-1}y}}{1+y^2}$$

which is a linear differential equation of the form

$$\frac{dx}{dy}+Px = Q$$

where, $P = \frac{1}{1+y^2}$ and $Q = \frac{e^{\tan^{-1}y}}{1+y^2}$

Now, $\text{IF} = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

and required solution is given by

$$x\cdot(\text{IF}) = \int (Q\times \text{IF})dy+C$$

$$\Rightarrow \quad x\cdot e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{(1+y^2)}\cdot e^{\tan^{-1}y}dy+C$$

$$\Rightarrow \quad x\cdot e^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{(1+y^2)}dy+C$$

Now, put $\tan^{-1}y = t \Rightarrow \frac{1}{1+y^2}dy = dt$

$$\therefore \quad x\cdot e^{\tan^{-1}y} = \int e^{2t}dt+C$$

$$\Rightarrow \quad x\cdot e^{\tan^{-1}y} = \frac{e^{2t}}{2}+C$$

$$\Rightarrow \quad x\cdot e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2}+C$$

$$\Rightarrow \quad x = \frac{e^{\tan^{-1}y}}{2}+Ce^{-\tan^{-1}y}$$

which is the required solution.

13. Given $\mathbf{a}+\mathbf{b}, \mathbf{b}+\mathbf{c}$ and $\mathbf{c}+\mathbf{a}$ are coplanar,

i.e. $[\mathbf{a}+\mathbf{b}\ \mathbf{b}+\mathbf{c}\ \mathbf{c}+\mathbf{a}] = 0$

To show $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanar, i.e. $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0$

Now, consider $[\mathbf{a}+\mathbf{b}\ \mathbf{b}+\mathbf{c}\ \mathbf{c}+\mathbf{a}] = 0$

$$\Rightarrow \quad (\mathbf{a}+\mathbf{b})\cdot\{(\mathbf{b}+\mathbf{c})\times(\mathbf{c}+\mathbf{a})\} = 0$$

$$\Rightarrow \quad (\mathbf{a}+\mathbf{b})\cdot\{(\mathbf{b}\times\mathbf{c})+(\mathbf{b}\times\mathbf{a})+(\mathbf{c}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})\} = 0$$

$$\Rightarrow \quad (\mathbf{a}+\mathbf{b})\cdot\{(\mathbf{b}\times\mathbf{c})+(\mathbf{b}\times\mathbf{a})+(\mathbf{c}\times\mathbf{a})\} = 0 \qquad [\because \mathbf{c}\times\mathbf{c} = 0]$$

$$\Rightarrow \quad \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})+\mathbf{a}\cdot(\mathbf{b}\times\mathbf{a})+\mathbf{a}\cdot(\mathbf{c}\times\mathbf{a})+0$$
$$\mathbf{b}\cdot(\mathbf{b}\times\mathbf{c})+\mathbf{b}\cdot(\mathbf{b}\times\mathbf{a})+\mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = 0$$

$$\Rightarrow \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]+0+0+0+0+[\mathbf{b}\ \mathbf{c}\ \mathbf{a}] = 0$$

$$\Rightarrow \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]+[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0 \qquad (\because [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = [\mathbf{b}\ \mathbf{c}\ \mathbf{a}] = [\mathbf{c}\ \mathbf{a}\ \mathbf{b}])$$

$$\Rightarrow \quad 2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0 \Rightarrow [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0 \qquad \textbf{Hence proved.}$$

14. Let the vector equation of required line be

$$\mathbf{r} = (\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}})+t(b_1\hat{\mathbf{i}}+b_2\hat{\mathbf{j}}+b_3\hat{\mathbf{k}}), \text{ where } t \text{ is a parameter.}$$

Since, the line is perpendicular to the two lines

$$\mathbf{r} = (8\hat{\mathbf{i}}-19\hat{\mathbf{j}}+10\hat{\mathbf{k}})+\lambda(3\hat{\mathbf{i}}-16\hat{\mathbf{j}}+7\hat{\mathbf{k}})$$

and $\mathbf{r} = (15\hat{\mathbf{i}}+29\hat{\mathbf{j}}+5\hat{\mathbf{k}})+\mu(3\hat{\mathbf{i}}+8\hat{\mathbf{j}}-5\hat{\mathbf{k}})$

Therefore, we have

$$3b_1-16b_2+7b_3 = 0 \qquad \ldots(i)$$

and $$3b_1+8b_2-5b_3 = 0 \qquad \ldots(ii)$$

Now, solving Eqs. (i) and (ii), we get the following

$$\frac{b_1}{80-56} = \frac{-b_2}{-15-21} = \frac{b_3}{24+48}$$

$$\Rightarrow \quad \frac{b_1}{24} = \frac{b_2}{36} = \frac{b_3}{72}$$

$$\Rightarrow \quad \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$$

Hence, the required vector equation is

$$\mathbf{r} = (\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}})+t(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}})$$

Now, for cartesian equation, put $\mathbf{r} = x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$

$$\Rightarrow \quad (x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}) = (1+2t)\hat{\mathbf{i}}+(2+3t)\hat{\mathbf{j}}+(-4+6t)\hat{\mathbf{k}}$$

$$\Rightarrow \quad x = 1+2t;\ y = 2+3t;\ z = -4+6t$$

$$\Rightarrow \quad \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

which is the required cartesian equation.

15. Let E_1 be the event that person A selected for job

E_2 be the event that person B selected for job

E_3 be the event that person C selected for job

and E be the event that the changes does not take place.

Then, $P(E_1) = \frac{1}{7}; P(E_2) = \frac{2}{7}; P(E_3) = \frac{4}{7},$

$$P\left(\frac{E}{E_1}\right) = 0.2;\ P\left(\frac{E}{E_2}\right) = 0.5 \text{ and } P\left(\frac{E}{E_3}\right) = 0.7$$

Now, required probability $= P\left(\frac{E_3}{E}\right)$

$$= \frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{4}{7}\cdot\frac{7}{10}}{\frac{1}{7}\cdot\frac{2}{10}+\frac{2}{7}\cdot\frac{5}{10}+\frac{4}{7}\cdot\frac{7}{10}} = \frac{28}{2+10+28} = \frac{28}{40} = \frac{7}{10} = 0.7$$

OR

Let E be the event that throwing a '7' in a single throw of a pair of dice, and F be the event that throwing a '10' in a single throw of a pair of dice. Then,

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

[$\because$ 7 can be thrown with a pair of dice in the following ways

(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)]

$\Rightarrow \quad P(\bar{E}) = \frac{5}{6}$ and $P(F) = \frac{3}{36} = \frac{1}{12}$

[$\because$ 10 can be thrown with a pair of dice in the following ways (4, 6), (6, 4), (5, 5)]

$\Rightarrow \quad P(\bar{F}) = \frac{11}{12}$

Now, B wins the game if he throw a '10' in 2nd or 4th or 6th .....throws.

$\therefore$ $P(B \text{ wins}) = P(\bar{E}F \text{ or } \bar{E}\bar{F}\bar{E}F \text{ or } \bar{E}\bar{F}\bar{E}\bar{F}\bar{E}F \text{ or} \ldots.)$

$= P(\bar{E}F) + P(\bar{E}\bar{F}\bar{E}F) + P(\bar{E}\bar{F}\bar{E}\bar{F}\bar{E}F) + \ldots..$

$= P(\bar{E}) \cdot P(F) + P(\bar{E}) \cdot P(\bar{F})\, P(\bar{E}) \cdot P(F) + P(\bar{E}) P(\bar{F})$

$P(\bar{E}) P(\bar{F}) P(\bar{E}) P(F) + \ldots$

$= \frac{5}{6} \cdot \frac{1}{12} + \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12} + \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12} + \ldots$

$= \frac{5}{72} + \left(\frac{5}{6} \cdot \frac{11}{12}\right) \cdot \frac{5}{72} + \left(\frac{5}{6} \cdot \frac{11}{12}\right)^2 \cdot \frac{5}{72} + \ldots$

$= \frac{\frac{5}{72}}{1 - \frac{5}{6} \cdot \frac{11}{12}} = \frac{\frac{5}{72}}{1 - \frac{55}{72}} = \frac{\frac{5}{72}}{\frac{17}{72}} = \frac{5}{17}$

16. Consider LHS $= \left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7}\right) + \left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{5} + \frac{1}{7}}{1 - \frac{1}{5} \cdot \frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{8}}{1 - \frac{1}{3} \cdot \frac{1}{8}}\right)$

$\left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right), \text{ if } xy < 1\right]$

$= \tan^{-1}\left(\frac{\left(\frac{12}{35}\right)}{\left(\frac{34}{35}\right)}\right) + \tan^{-1}\left(\frac{\left(\frac{11}{24}\right)}{\left(\frac{23}{24}\right)}\right) = \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right)$

$= \tan^{-1}\left[\frac{\frac{12}{34} + \frac{11}{23}}{1 - \frac{12}{34} \cdot \frac{11}{23}}\right] = \tan^{-1}\left(\frac{\frac{276 + 374}{34 \cdot 23}}{\frac{782 - 132}{34 \cdot 23}}\right)$

$= \tan^{-1}\left(\frac{650}{650}\right) = \tan^{-1}(1) = \frac{\pi}{4}$ **Hence proved.**

OR

Consider, the given equation

$2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$\Rightarrow \quad \tan^{-1}\left(\frac{2 \cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x)$

$\left[\because 2 \tan^{-1} x = \tan^{-1}\left(\frac{2x}{1 - x^2}\right), \text{ if } -1 \le x \le 1\right]$

$\Rightarrow \quad \tan^{-1}\left(\frac{2 \cos x}{\sin^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x)$

$\Rightarrow \quad \frac{2 \cos x}{\sin^2 x} = 2 \operatorname{cosec} x \quad [\because \tan(\tan^{-1} x) = x, \forall x \in R]$

$\Rightarrow \quad \cos x = \frac{1}{\sin x} \cdot \sin^2 x$

$\Rightarrow \quad \cos x = \sin x$

$\Rightarrow \quad \tan x = 1$

$\Rightarrow \quad x = \frac{\pi}{4}$

17. Let the incomes of Aryan and Babban are ₹ $3x$ and ₹ $4x$, respectively and their expenditures are ₹ $5y$ and ₹ $7y$, respectively. Then, according to given information, we have

$3x - 5y = 15000 \quad \ldots(i)$

$4x - 7y = 15000 \quad \ldots(ii)$

Above system of equations can be written in matrix form as

$AX = B \quad \ldots(iii)$

where, $A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

Clearly, $A^{-1} = \frac{1}{|A|}\begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$

$\left[\because \text{if } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is non-singular, then } A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right]$

$= \frac{1}{(-21 + 20)}\begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$

Now, from Eq. (iii), we have

$X = A^{-1}B = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000\,(7 - 5) \\ 15000\,(4 - 3) \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 15000 \end{bmatrix}$

$\Rightarrow \quad x = 30000 \text{ and } y = 15000$

Thus, Aryan's income = ₹ 90000

and Babban's income = ₹ 120000

Clearly, this problem reflects the value of saving.

18. We have, $x = a \sin 2t (1 + \cos 2t)$

Clearly, $\frac{dx}{dt} = a[\sin 2t\,(-\sin 2t \cdot 2) + (1 + \cos 2t) \cdot \cos 2t \cdot 2]$

$= 2a[\cos 2t + \cos^2 2t - \sin^2 2t]$

$= 2a[\cos 2t + \cos 4t]$

$= 4a \cos 3t \cos t \quad \ldots(i)$

and $y = b \cos 2t\,(1 - \cos 2t)$

$\Rightarrow \quad \frac{dy}{dt} = b[\cos 2t \cdot \sin 2t \cdot 2 + (1 - \cos 2t)(-\sin 2t) \cdot 2]$

$= 2b[\cos 2t \sin 2t - \sin 2t + \sin 2t \cos 2t]$

$= 2b[2 \sin 2t \cos 2t - \sin 2t]$

$= 2b[\sin 4t - \sin 2t]$

$= 4b \cos 3t \sin t \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4b \cos 3t \sin t}{4a \cos 3t \cos t} \Rightarrow \frac{dy}{dx} = \frac{b}{a} \tan t$

$\Rightarrow \quad \left[\frac{dy}{dx}\right]_{\text{at } t = \frac{\pi}{4}} = \frac{b}{a}$ and $\left[\frac{dy}{dx}\right]_{\text{at } t = \frac{\pi}{3}} = \sqrt{3}\,\frac{b}{a}$

OR

Given $y = x^x$

To prove $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

Consider, $y = x^x$

On taking log both sides, we get

$\log y = \log x^x \Rightarrow \log y = x \log x$

Now, on differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx}=x\cdot\frac{1}{x}+\log x$$

$$\Rightarrow \quad \frac{1}{y}\cdot\frac{dy}{dx}=1+\log x \qquad \ldots(i)$$

$$\Rightarrow \quad \frac{dy}{dx}=y(1+\log x)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=y\cdot\frac{1}{x}+(1+\log x)\cdot\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=\frac{y}{x}+\frac{1}{y}\left(\frac{dy}{dx}\right)^2 \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0 \qquad \textbf{Hence proved.}$$

19. Given, $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2}\\ p, & \text{if } x=\dfrac{\pi}{2}\\ \dfrac{q(1-\sin x)}{(\pi-2x)^2}, & \text{if } x>\dfrac{\pi}{2}\end{cases}$

is continuous at $x=\dfrac{\pi}{2}$.

$$\therefore \quad \lim_{x\to\frac{\pi}{2}^+}f(x)=\lim_{x\to\frac{\pi}{2}^-}f(x)=f\left(\frac{\pi}{2}\right)=p \qquad \ldots(i)$$

Consider, LHL $=\lim\limits_{x\to\frac{\pi}{2}^-}f(x)=\lim\limits_{h\to 0}f\left(\frac{\pi}{2}-h\right)$

$$=\lim_{h\to 0}\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)}=\lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h}$$

$$=\frac{1}{3}\lim_{h\to 0}\frac{(1-\cos h)(1+\cos^2 h+\cos h)}{\sin^2 h}$$

$$=\frac{1}{3}\lim_{h\to 0}\frac{1-\cosh}{\sin^2 h}\cdot\lim_{h\to 0}(1+\cos^2 h+\cos h)$$

$$=\frac{\lim\limits_{h\to 0}\left(\frac{1-\cos h}{h^2}\right)}{\lim\limits_{h\to 0}\left(\frac{\sin h}{h}\right)^2}=\lim_{h\to 0}\frac{1-\cos h}{h^2}=\lim_{h\to 0}\frac{2\sin^2\left(\frac{h}{2}\right)}{4\left(\frac{h}{2}\right)^2}=\frac{1}{2} \qquad \ldots(ii)$$

Now consider, RHL $=\lim\limits_{x\to\frac{\pi}{2}^+}f(x)$

$$=\lim_{h\to 0}f\left(\frac{\pi}{2}+h\right)=\lim_{h\to 0}\frac{q\left\{1-\sin\left(\frac{\pi}{2}+h\right)\right\}}{\left\{\pi-2\left(\frac{\pi}{2}+h\right)\right\}^2}$$

$$=\lim_{h\to 0}\frac{q(1-\cos h)}{(\pi-\pi-2h)^2}=\lim_{h\to 0}\frac{q(1-\cos h)}{4h^2}$$

$$=\frac{q}{4}\lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2}=\frac{q}{4}\lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{4\cdot\left(\frac{h}{2}\right)^2}$$

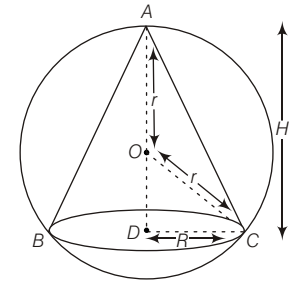
$$=\frac{q}{8}\lim_{h\to 0}\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\}^2=\frac{q}{8}\cdot 1=\frac{q}{8} \qquad \ldots(iii)$$

Thus, from Eqs. (i), (ii) and (iii), we get

$$\frac{q}{8}=\frac{1}{2}=p \quad \Rightarrow \quad p=\frac{1}{2} \text{ and } q=4$$

20. Let R and H be the radius and altitude of cone inscribed in a sphere of radius r. Then, volume of cone

$$=V=\frac{1}{3}\pi R^2H \qquad \ldots(i)$$



Now, from ΔODC, we have

$$OC^2=OD^2+DC^2$$

$$\Rightarrow \quad r^2=(H-r)^2+R^2$$

$$\Rightarrow \quad R^2=r^2-(H-r)^2=r^2-(H^2+r^2-2Hr)$$

$$=2Hr-H^2$$

From Eq. (i), we get

$$V=V(H)=\frac{1}{3}\pi(2Hr-H^2)\cdot H$$

$$=\frac{\pi}{3}(2H^2r-H^3) \qquad \ldots(ii)$$

$$\Rightarrow \quad V'(H)=\frac{\pi}{3}(4Hr-3H^2)$$

For maximum, put $V'(H)=0$

$$\Rightarrow \quad H(4r-3H)=0 \quad \Rightarrow \quad H=0 \text{ or } H=\frac{4r}{3}$$

$$\Rightarrow \quad H=\frac{4r}{3} \qquad [\because H\neq 0]$$

Now, $\quad V''(H)=\dfrac{\pi}{3}(4r-6H)$

and $\quad V''\left(\dfrac{4r}{3}\right)=\dfrac{\pi}{3}\left(4r-6\cdot\dfrac{4r}{3}\right)=\dfrac{-4r\pi}{3}<0$.

Hence, by second derivative test, the volume is maximum when

$$H=\frac{4r}{3}$$

Now, from Eq. (ii), we have

$$V=\frac{\pi}{3}(2H^2r-H^3)$$

$$=\frac{\pi}{3}\left\{2\cdot\left(\frac{4r}{3}\right)^2\cdot r-\left(\frac{4r}{3}\right)^3\right\} \qquad \left[\because H=\frac{4r}{3}\right]$$

$$=\frac{\pi}{3}\left(\frac{32r^3}{9}-\frac{64r^3}{27}\right)=\frac{32\pi r^3}{27}\left(1-\frac{2}{3}\right)$$

$$=\frac{32\pi}{27}r^3\left(\frac{1}{3}\right)=\frac{32\pi}{81}r^3$$

$$=\frac{8}{27}\left(\frac{4}{3}\pi r^3\right)=\frac{8}{27} \qquad \text{(volume of sphere)}$$

Hence, the volume of the largest cone that can be inscribed in the sphere is $\dfrac{8}{27}$ of the volume of sphere.

OR

Given, $f(x) = \sin 3x - \cos 3x,\ 0 < x < \pi$

$\Rightarrow$ $f'(x) = 3\cos 3x + 3\sin 3x$

$\Rightarrow$ $f'(x) = 3(\sin 3x + \cos 3x)$

$$= 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x\right)$$

$$= 3\sqrt{2}\left(\cos\frac{\pi}{4}\sin 3x + \sin\frac{\pi}{4}\cos 3x\right)$$

$$= 3\sqrt{2}\sin\left(3x + \frac{\pi}{4}\right)$$

Now, $f'(x) = 0 \Rightarrow 3\sqrt{2}\sin\left(3x + \frac{\pi}{4}\right) = 0$

$\Rightarrow$ $\sin\left(3x + \frac{\pi}{4}\right) = 0 \Rightarrow 3x + \frac{\pi}{4} = n\pi,\ n \in Z$

$\Rightarrow$ $3x + \frac{\pi}{4} = \pi, 2\pi, 3\pi \left[\because 0 < x < \pi \ \therefore \frac{\pi}{4} < 3x + \frac{\pi}{4} < 13\frac{\pi}{4}\right]$

$\Rightarrow$ $x = \frac{\pi}{4}, \frac{7\pi}{12}$ and $\frac{11\pi}{12}$

Thus, the points $x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$ divides the interval $(0, \pi)$ into four disjoints intervals

$$\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{7\pi}{12}\right), \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right) \text{and} \left(\frac{11\pi}{12}, \pi\right)$$

Note that,

(i) if $0 < x < \frac{\pi}{4} \Rightarrow \frac{\pi}{4} < 3x + \frac{\pi}{4} < \pi \Rightarrow f'(x) > 0$

(ii) if $\frac{\pi}{4} < x < \frac{7\pi}{12} \Rightarrow \pi < 3x + \frac{\pi}{4} < 2\pi \Rightarrow f'(x) < 0$

(iii) if $\frac{7\pi}{12} < x < \frac{11\pi}{12} \Rightarrow 2\pi < 3x + \frac{\pi}{4} < 3\pi \Rightarrow f'(x) > 0$

(iv) if $\frac{11\pi}{12} < x < \pi \Rightarrow 3\pi < 3x + \frac{\pi}{4} < \frac{13\pi}{4} \Rightarrow f'(x) < 0$

Thus, f is strictly increasing in the intervals $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ and strictly decreasing in the intervals $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right)$ and $\left(\frac{11\pi}{12}, \pi\right)$.

21. Given equations of curves are

$$x^2 + y^2 = 2ax \quad \ldots(i)$$

and $$y^2 = ax \quad \ldots(ii)$$

Clearly, Eq. (i) is a circle with centre $(a, 0)$ and radius $= \sqrt{a^2 + 0^2 - 0} = a$ and Eq. (ii) is a parabola with vertex $(0, 0)$ and axis parallel to X-axis.

Now, let us first find the intersection points of given curves.

On substituting $y^2 = ax$ in Eq. (i), we get

$$x^2 + ax = 2ax$$

$\Rightarrow$ $x^2 - ax = 0$

$\Rightarrow$ $x(x - a) = 0$

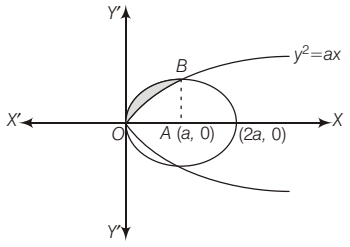
$\Rightarrow$ $x = 0$ or $x = a$

and from Eq. (ii), we get

$$y = 0 \text{ or } y = \pm a$$

Thus, intersection points are $(0, 0)$, (a, a) and $(a, -a)$

Now, let us draw the graphs of given curves.



Clearly required area = area of shaded region

$$= \frac{\text{Area of circle}}{4} - \int_0^a y\,(\text{Parabola})\,dx$$

$$= \frac{\pi a^2}{4} - \int_0^a \sqrt{ax}\,dx$$

$$= \frac{\pi a^2}{4} - \sqrt{a}\int_0^a x^{1/2}\,dx$$

$$= \frac{\pi a^2}{4} - \sqrt{a} \times \frac{2}{3}[x^{3/2}]_0^a$$

$$= \frac{\pi a^2}{4} - \frac{2}{3}\sqrt{a} \cdot a^{3/2}$$

$$= \frac{\pi a^2}{4} - \frac{2}{3}a^2 = \left(\frac{\pi}{4} - \frac{2}{3}\right)a^2$$

22. Clearly, equation of the AB is

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \quad \ldots(i)$$

and equation of plane is given by $\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$

$\Rightarrow$ $(x-2)(2-0) - (y-2)(-1-0) + (z-1)(-3+4) = 0$

$\Rightarrow$ $2(x-2) + (y-2) + (z-1) = 0$

$\Rightarrow$ $2x + y + z - 7 = 0 \quad \ldots(ii)$

Now, let

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$$

$\Rightarrow$ $x = 3 - \lambda,\ y = \lambda - 4,\ z = 6\lambda - 5$

Thus, the coordinate of P will be $(3 - \lambda, \lambda - 4, 6\lambda - 5)$.

Since, P lies on the plane, therefore from Eq. (ii), we have

$$2(3 - \lambda) + \lambda - 4 + 6\lambda - 5 - 7 = 0$$

$\Rightarrow$ $6 - 2\lambda + \lambda - 4 + 6\lambda - 12 = 0$

$\Rightarrow$ $5\lambda - 10 = 0$

$\Rightarrow$ $\lambda = 2$

$\Rightarrow$ Coordinates of point P are $(1, -2, 7)$.

Now, let P divides AB in the ratio $k : 1$.

$\therefore$ $1 = \frac{2.k + 3}{k + 1} \Rightarrow k + 1 = 2k + 3 \Rightarrow -2 = k$

Hence, P divides AB externally in the ratio 2:1.

23. Let p = Probability of success, i.e. getting a red ball

q = Probability of failure, i.e. getting a white ball

X = Random variable that denotes the number of success in four drawn.

Clearly, X follows binomial distribution with

$$p = \frac{6}{9} = \frac{2}{3}; q = \frac{3}{9} = \frac{1}{3} \text{ and } n = 4$$

Thus, $P(X=x)={}^4C_x\left(\frac{1}{3}\right)^{4-x}\left(\frac{2}{3}\right)^x$, where $x=0,1,2,3,4$

Now, $P(X=0)={}^4C_0\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0=\frac{1}{81}$

$P(X=1)={}^4C_1\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1=\frac{8}{81}$

$P(X=2)={}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2=\frac{24}{81}$

$P(X=3)={}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3=\frac{32}{81}$

and $P(X=4)={}^4C_4\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^4=\frac{16}{81}$

Thus, the probability distribution of X is

X	0	1	2	3	4
$P(X)$	$\frac{1}{81}$	$\frac{8}{81}$	$\frac{24}{81}$	$\frac{32}{81}$	$\frac{16}{81}$

Now, mean of distribution $=E(X)=\Sigma\, x\cdot P(X=x)$

$=0\cdot\frac{1}{81}+1\cdot\frac{8}{81}+2\cdot\frac{24}{81}+3\cdot\frac{32}{81}+4\cdot\frac{16}{81}$

$=\frac{1}{81}[8+48+96+64]=\frac{216}{81}=\frac{8}{3}$

and variance of distribution $=E(X^2)-(E(X))^2$

$=\Sigma\, x^2\cdot P(X=x)-\left(\frac{8}{3}\right)^2$

$=\left[0^2\cdot\frac{1}{81}+1^2\cdot\frac{8}{81}+2^2\cdot\frac{24}{81}+3^2\cdot\frac{32}{81}+4^2\cdot\frac{16}{81}\right]-\left(\frac{8}{3}\right)^2$

$=\frac{1}{81}[0+8+96+288+256]-\frac{64}{9}=8-\frac{64}{9}=\frac{72-64}{9}=\frac{8}{9}$

24. Let the manufacturer produces x units of product A and y units of product B, per day. Then, the given problem can be formulated as follows

Maximize, $Z=7x+4y$

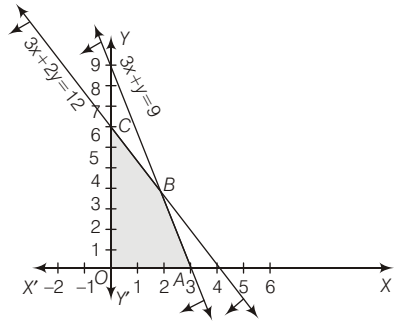
Subject to the constraints,

$3x+2y\le 12$ ...(i)

$3x+y\le 9$ ...(ii)

$x, y\ge 0$ ...(iii)

The feasible region determined by the constraints is as follows



Clearly, the corner points of the feasible region are O, A, B and C and the coordinates of O, A and C are (0, 0), (3, 0) and (0, 6) respectively.

To find the coordinates of point B let us solve the following equations

$3x+2y=12$ ...(iv)

and $3x+y=9$ ...(v)

On solving Eqs. (iv) and (v), we get

$x=2$ and $y=3$

Thus, the coordinates of B are (2, 3).

Now, let us evaluate the values of objective function $Z=7x+4y$ at the corner points of the feasible region, which is given in the following table

Corner points	Value of $Z=7x+4y$
$O(0, 0)$	0
$A(3, 0)$	21
$B(2, 3)$	26
$C(0, 6)$	24

Clearly, Z is maximum when $x=2$ and $y=3$.

Hence, the manufacturer must produce 2 units of product A and 3 units of product B per day, to maximize the profit.

25. **Given** A function $f: N\to N$ defined as $f(x)=9x^2+6x-5$.

To show $f: N\to S$, where S is range of f, is invertible.

Since, $S=$ Range f, therefore f is onto .

Now, let $x_1, x_2\in N$ such that $f(x_1)=f(x_2)$

$\Rightarrow\quad 9x_1^2+6x_1-5=9x_2^2+6x_2-5$

$\Rightarrow\quad 9(x_1^2-x_2^2)+6(x_1-x_2)=0$

$\Rightarrow\quad 9(x_1+x_2)(x_1-x_2)+6(x_1-x_2)=0$

$\Rightarrow\quad (x_1-x_2)[9(x_1+x_2)+6]=0$

$\Rightarrow\quad x_1-x_2=0$ [$\because x_1, x_2\in N$, therefore $9x_1+9x_2+6\ne 0$]

$\Rightarrow\quad x_1=x_2$

$\Rightarrow f$ is one-one

Hence $f: N\to S$ is invertible.

Now to find f^{-1}, let $y=9x^2+6x-5$

$\Rightarrow\quad 9x^2+6x-(5+y)=0$

$\Rightarrow\quad x=\frac{-6\pm\sqrt{36+4\times 9(5+y)}}{18}$

$\Rightarrow\quad x=\frac{-6\pm 6\sqrt{1+(5+y)}}{18}$

$\Rightarrow\quad x=\frac{-1\pm\sqrt{6+y}}{3}\Rightarrow x=\frac{-1+\sqrt{6+y}}{3}$ [$\because x\in N$]

Thus, $f^{-1}(x)=\frac{-1+\sqrt{6+y}}{3}$

Also, $f^{-1}(43)=\frac{-1+\sqrt{6+43}}{3}=\frac{-1+7}{3}=2$

and $f^{-1}(163)=\frac{-1+\sqrt{6+163}}{3}=\frac{-1+13}{3}=4$

26. Let $\Delta=\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2\\ zx-y^2 & xy-z^2 & yz-x^2\\ xy-z^2 & yz-x^2 & zx-y^2\end{vmatrix}$

On applying $R_1\to R_1+R_2+R_3$, we get

$\Delta=\begin{vmatrix} (xy+yz+zx-x^2-y^2-z^2) & (xy+yz+zx-x^2-y^2-z^2) & (xy+yz+zx-x^2-y^2-z^2)\\ zx-y^2 & xy-z^2 & yz-x^2\\ xy-z^2 & yz-x^2 & zx-y^2\end{vmatrix}$

Taking common $(xy+yz+zx-x^2-y^2-z^2)$ from R_1, we get

$$\Delta=(xy+yz+zx-x^2-y^2-z^2)\begin{vmatrix}1 & 1 & 1\\ zx-y^2 & xy-z^2 & yz-x^2\\ xy-z^2 & yz-x^2 & zx-y^2\end{vmatrix}$$

On applying $C_2\to C_2-C_1$ and $C_3\to C_3-C_1$, we get

$$\Delta=(xy+yz+zx-x^2-y^2-z^2)\begin{vmatrix}1 & 0 & 0\\ zx-y^2 & x(y-z)-(z^2-y^2) & z(y-x)-(x^2-y^2)\\ xy-z^2 & y(z-x)-(x^2-z^2) & x(z-y)-(y^2-z^2)\end{vmatrix}$$

$$\Rightarrow\ \Delta=(xy+yz+zx-x^2-y^2-z^2)\begin{vmatrix}1 & 0 & 0\\ zx-y^2 & x(y-z)-(z+y)(z-y) & z(y-x)-(x-y)(x+y)\\ xy-z^2 & y(z-x)-(x-z)(x+z) & x(z-y)-(y-z)(y+z)\end{vmatrix}$$

$$\Delta=(xy+yz+zx-x^2-y^2-z^2)\begin{vmatrix}1 & 0 & 0\\ zx-y^2 & (y-z)(x+y+z) & (y-x)(x+y+z)\\ xy-z^2 & (z-x)(x+y+z) & (z-y)(x+y+z)\end{vmatrix}$$

Taking common $(x+y+z)$ from C_2 and C_3, we get

$$\Delta=(x+y+z)^2(xy+yz+zx-x^2-y^2-z^2)\begin{vmatrix}1 & 0 & 0\\ zx-y^2 & y-z & y-x\\ xy-z^2 & z-x & z-y\end{vmatrix}$$

Now, on expanding along R_1, we get

$$\Delta=(x+y+z)^2(xy+yz+zx-x^2-y^2-z^2)\,1\cdot[(y-z)(z-y)-(y-x)(z-x)]$$
$$=(x+y+z)^2(xy+yz+zx-x^2-y^2-z^2)\,[-(z-y)^2-yz+xy+xz-x^2]$$
$$=(x+y+z)^2(xy+yz+zx-x^2-y^2-z^2)\,[xy+yz+zx-x^2-y^2-z^2]$$
$$=(x+y+z)^2(xy+yz+zx-x^2-y^2-z^2)^2$$

Clearly, Δ is divisible by $(x+y+z)$ and the quotient is $(x+y+z)(xy+yz+zx-x^2-y^2-z^2)^2$

OR

Given, $A=\begin{pmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}$

Consider, $A=IA\ \Rightarrow\begin{pmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$

On applying $R_3\leftrightarrow R_1$, we get

$$\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 1\\ 8 & 4 & 3\end{pmatrix}=\begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}A$$

On applying $R_2\to R_2-2R_1$ and $R_3\to R_3-8R_1$, we get

$$\begin{pmatrix}1 & 2 & 2\\ 0 & -3 & -3\\ 0 & -12 & -13\end{pmatrix}=\begin{pmatrix}0 & 0 & 1\\ 0 & 1 & -2\\ 1 & 0 & -8\end{pmatrix}A$$

On applying $R_2\to-\frac{1}{3}R_2$, we get

$$\begin{pmatrix}1 & 2 & 2\\ 0 & 1 & 1\\ 0 & -12 & -13\end{pmatrix}=\begin{pmatrix}0 & 0 & 1\\ 0 & -1/3 & 2/3\\ 1 & 0 & -8\end{pmatrix}A$$

On applying $R_3\to R_3+12R_2$, we get

$$\begin{pmatrix}1 & 2 & 2\\ 0 & 1 & 1\\ 0 & 0 & -1\end{pmatrix}=\begin{pmatrix}0 & 0 & 1\\ 0 & -1/3 & 2/3\\ 1 & -4 & 0\end{pmatrix}A$$

On applying $R_2\to R_2+R_3$ and $R_1\to R_1+2R_3$, we get

$$\begin{pmatrix}1 & 2 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix}=\begin{pmatrix}2 & -8 & 1\\ 1 & -13/3 & 2/3\\ 1 & -4 & 0\end{pmatrix}A$$

On applying $R_3\to(-1)R_3$, we get

$$\begin{pmatrix}1 & 2 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}2 & -8 & 1\\ 1 & -13/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}A$$

On applying $R_1\to R_1-2R_2$ we get

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 2/3 & -1/3\\ 1 & -13/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}A$$

Thus, $A^{-1}=\begin{pmatrix}0 & 2/3 & -1/3\\ 1 & -13/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}$

Now, consider the given system of equations

$$8x+4y+3z=19,\quad 2x+y+z=5,\quad x+2y+2z=7$$

which can be written in matrix form as $AX=B$

where, $A=\begin{bmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

and $B=\begin{bmatrix}19\\ 5\\ 7\end{bmatrix}$

Clearly, $X=A^{-1}B$

$$=\begin{pmatrix}0 & 2/3 & -1/3\\ 1 & -13/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}\begin{pmatrix}19\\ 5\\ 7\end{pmatrix}$$

$$=\begin{pmatrix}0+\frac{10}{3}-\frac{7}{3}\\ 19-\frac{65}{3}+\frac{14}{3}\\ -19+20\end{pmatrix}=\begin{pmatrix}1\\ 2\\ 1\end{pmatrix}$$

Hence, $x=1,\ y=2$ and $z=1$

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MATRICES

MATRIX

A rectangular array of mn numbers in the form of m horizontal lines (called rows) and n vertical lines (called columns), is called a matrix of order m by n, written as $m \times n$ matrix. Such an array is enclosed by [ ] or ( ). An $m \times n$ matrix is usually written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

In compact form, the above matrix is represented by $A = [a_{ij}]_{m \times n}$. The numbers $a_{11}, a_{12}, \dots,$ etc., are known as elements of the matrix A, a_{ij} belong to the ith row and jth column and is called (i, j)th element of the matrix A.

CLASSIFICATION OF MATRICES

1. **Row matrix** A matrix having only one row, is called a row matrix or row vector.
2. **Column matrix** A matrix having only one column, is called a column matrix or a column vector.
3. **Horizontal matrix** If in a matrix, the number of rows is less than the number of columns, then it is called a horizontal matrix.
4. **Vertical matrix** If in a matrix, the number of rows is greater than number of columns, then it is called the vertical matrix.
5. **Zero matrix or null matrix** A matrix each of whose elements are zero, is called zero matrix or a null matrix.
6. **Square matrix** A matrix in which number of rows is equal to the number of columns, say m, is called square matrix of order m.
7. **Diagonal matrix** A square matrix $A = [a_{ij}]_{n \times n}$ is called diagonal matrix, if all the elements except in the leading diagonal are zero, i.e. $a_{ij} = 0$ for $i \neq j$. It is written as $A = \text{diag}\,(a_{11}, a_{22}, \dots, a_{nn})$.
8. **Scalar matrix** A square matrix in which every non-diagonal elements are zero and all diagonal elements are equal, is known as a scalar matrix.
9. **Unit matrix** A square matrix in which every non-diagonal elements are zero and every diagonal elements are 1, is called a unit matrix or an identity matrix. Thus, a square matrix $A = [a_{ij}]_{n \times n}$ is unit matrix, if $a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ 1, & \text{when } i = j \end{cases}$

   A unit matrix of order n is denoted by I_n or I.
10. **Upper triangular matrix** A square matrix $A = [a_{ij}]$ is called upper triangular matrix, if $a_{ij} = 0, \forall\, i > j$.
11. **Lower triangular matrix** A square matrix $A = [a_{ij}]$ is called lower triangular matrix, if $a_{ij} = 0, \forall\, i < j$.
12. **Singular and non-singular matrix** A square matrix A is said to be non-singular, if $|A| \neq 0$ and a square matrix A is said to be singular, if $|A| = 0$.

TRACE OF A MATRIX

The sum of the diagonal elements of a square matrix A is called the trace of A and is denoted by tr (A).

EQUALITY OF MATRICES

Two matrices A and B are said to be equal, if they are of same order and all corresponding elements are equal. It is written as $A = B$.

ADDITION OF MATRICES

Let A and B be two matrices. Then, the sum of matrices defined only, if A and B are of same order. Thus, if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two matrices of same order, then the sum $A + B$ is defined and $A + B = [a_{ij} + b_{ij}]_{m \times n}$.

PROPERTIES OF ADDITION OF MATRICES

(i) **Matrix addition is commutative** If A and B are two matrices of the same order, then $A + B = B + A$.

(ii) **Matrix addition is associative** If A, B and C are three matrices of same order, then $(A + B) + C = A + (B + C)$.

(iii) **Existence of additive identity** If O is null matrix of same order as that of matrix A, then $A + O = O + A = A$.

(iv) **Existence of additive inverse** If A is any matrix, then $A + (-A) = O = (-A) + A$.

(v) **Cancellation laws** If A, B and C are matrices of same order, then

$A + B = A + C \Rightarrow B = C$ [left cancellation law]

$B + A = C + A \Rightarrow B = C$ [right cancellation law]

Inverse of a Squ... Elementary Tr...

INVERSE OF A SQUARE MATRIX

Let A be any n-rowed square matrix. Then, a square matrix B, such that $AB = BA = I$, is called inverse of A. It is denoted by A^{-1}. It may be noted that $AA^{-1} = A^{-1}A = I$

ELEMENTARY TRANSFORMATIONS

Each one of the following operations on a matrix is called an elementary transformation.

(i) Interchanging any two rows or columns. This transformation is indicated by $R_i \leftrightarrow R_j (C_i \leftrightarrow C_j)$

(ii) Multiplication of elements of any row or column by a non-zero scalar quantity k. This transformation is indicated by $R_i \to kR_i (C_i \to kC_i)$.

(iii) Addition of constant multiple $k (k \neq 0)$ of the elements of any row (or column) to the corresponding element of any other row (or column). This transformation is indicated by $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

SUBTRACTION OF MATRICES

Let A and B be two matrices of the same order, then subtraction of two matrices $(A - B)$ is basically sum of matrix A with additive inverse of the matrix B, i.e. $(-B)$. We can obtained the elements of $A - B$ by subtracting the elements of B from A.

MULTIPLICATION OF MATRIX BY SCALAR

Let $A = [a_{ij}]$ be an $m \times n$ matrix and k be any scalar. The matrix obtained by multiplying each element of A by k is called the scalar multiple of A by k and is denoted by kA. Thus, if $A = [a_{ij}]_{m \times n}$, then $kA = [ka_{ij}]_{m \times n}$.

IMPORTANT POINTS TO MEMORISE

- No element of principal diagonal in diagonal matrix is zero.
- Number of zeroes in diagonal matrix is given by $n^2 - n$, where n is the order of the matrix.
- A matrix is said to be triangular, if it is upper triangular or lower triangular.
- Diagonal matrix is both upper and lower triangular.
- $\text{diag}\,(a_1, a_2, \dots a_n) \times \text{diag}\,(b_1, b_2, \dots, b_n) = \text{diag}\,(a_1 b_1, a_2 b_2, a_3 b_3, \dots, a_n b_n)$
- Multiplication of two scalar matrices is also a scalar matrix.
- Minimum number of zeroes in a triangular matrix is given by $\frac{n(n-1)}{2}$, where n is order of matrix.
- A triangular matrix $A = [a_{ij}]_{n \times n}$ is strictly triangular, if $a_{ii} = 0$ for $1 \leq i \leq n$.
- The multiplication of two triangular matrices is a triangular matrix.
- If A and B are two square matrices of the same order, say n, then both the products AB and BA are defined and each is a square matrix of order n.
- In the matrix product AB, the matrix A is called pre-multiplier (pre-factor) and B is called post-multiplier (post-factor).
- The rule of multiplication of matrices is row-columnwise $(\to \downarrow)$, i.e. the first row of AB is obtained by multiplying the first row of A with first, second, third, ... columns of B, respectively. Similarly, second row of AB is obtained by multiplying the second row of A with first, second, third,... columns of B respectively and so on.
- $AB = O$ (null matrix) does not necessarily imply that $A = O$ or $B = O$ or $A, B = O$.

MULTIPLICATION OF MATRICES

Two matrices A and B can be multiplied only, if number of columns in A is same as the number of rows in B. e.g. If $A=[a_{ij}]_{m\times n}$ and $B=[b_{jk}]_{n\times p}$ are two matrices of order $m\times n$ and $n\times p$, respectively. Then, their product AB is of order $m\times p$.

$$\underbrace{A_{m\times n}}_{\text{Number of columns of } A \text{ = Number of rows of } B}\cdot\underbrace{B_{n\times p}}_{}=\underbrace{AB_{m\times p}}_{\text{Order of resultant matrix}}$$

PROPERTIES OF MATRIX MULTIPLICATION

(i) **Matrix multiplication is distributive over matrix addition** If A, B and C are the matrices of order $m\times n$, $n\times p$ and $n\times p$ respectively, then $A(B+C)=AB+AC$.

(ii) **Matrix multiplication is associative** If A, B and C are matrices of order $m\times n$, $n\times p$ and $p\times r$ respectively, then $(AB)C=A(BC)$.

(iii) **Existence of multiplicative identity** If A is an $m\times n$ matrix, I_n is the identity matrix of order $n\times n$ and I_m is the identity matrix of order $m\times m$, then $I_m A=A$ and $AI_n=A$. In particular, if A is a square matrix of order n, then $AI_n=I_n A=A$.

TRANSPOSE OF A MATRIX

Let A be an $m\times n$ matrix. Then, the $n\times m$ matrix obtained by interchanging the rows and columns of A, is called the transpose of A and is denoted by A' or A^T or A^C, thus

(i) If order of A is $m\times n$, then order of A^T is $n\times m$.

(ii) (i,j)th element of $A=(j,i)$th element of A^T.

PROPERTIES OF TRANSPOSE OF A MATRIX

(i) $(A^T)^T=A$ (ii) $(A+B)^T=A^T+B^T$

(iii) $(AB)^T=B^T A^T$ (iv) $(kA)^T=kA^T$

(v) $(A^n)^T=(A^T)^n$ (vi) $\text{Trace}(A^T)=\text{Trace}(A)$

(vii) $\text{Trace}(AA^T)\geq 0$

CONJUGATE OF A MATRIX

The matrix obtained from any given matrix A by taking conjugate of its each elements is called conjugate of A and is denoted by $\overline{A}$.

PROPERTIES OF CONJUGATE

(i) If A is any matrix, then $\overline{(\overline{A})}=A$.

(ii) If A and B are matrices of same order, then $\overline{(A+B)}=\overline{A}+\overline{B}$.

(iii) If A is any $m\times n$ and B is any $n\times p$ matrix, then $\overline{(AB)}=\overline{A}\cdot\overline{B}$.

(iv) If A is any matrix and k is any scalar, then $\overline{(kA)}=k\,\overline{A}$.

(v) If A is any square matrix, then $\overline{(A^n)}=(\overline{A})^n$.

TRANSPOSE CONJUGATE OF A MATRIX

The transpose of the conjugate of a matrix A is called transpose conjugate of A and is denoted by A^θ or A^*.

PROPERTIES OF TRANSPOSE CONJUGATE

(i) $(\overline{A'})=(\overline{A})'=A^\theta$ (ii) $(A^\theta)^\theta=A$

(iii) $(A+B)^\theta=A^\theta+B^\theta$

(iv) $(k\,A)^\theta=\overline{k}\,A^\theta$, k being any number

(v) $(AB)^\theta=B^\theta A^\theta$ (vi) $(A^n)^\theta=(A^\theta)^n$

SOME SPECIAL MATRICES

Symmetric Matrix A square matrix A is said to be symmetric, if $A'=A$, i.e. the matrix $A=[a_{ij}]_{n\times n}$ is symmetric, if $a_{ij}=a_{ji}$ for all i and j.

Skew-symmetric Matrix A square matrix A is said to be skew-symmetric, if $A^T=-A$,

i.e. the matrix $A=[a_{ij}]_{m\times n}$ is skew-symmetric, if $a_{ij}=-a_{ji}$ for all i and j.

Orthogonal Matrix A square matrix A is said to be orthogonal, if $AA^T=I=A^T A$.

PROPERTIES OF ORTHOGONAL MATRIX

(i) If A is an orthogonal matrix, then A^T is also orthogonal.

(ii) For any two orthogonal matrices A and B of some order AB and BA are orthogonal matrices.

(iii) If A is an orthogonal matrix, then A^{-1} is also orthogonal.

Unitary Matrix A square matrix is said to be unitary, if $\overline{A}'A=I$.

Hermitian Matrix A square matrix $A=[a_{ij}]$ is said to be hermitian matrix, if $a_{ij}=\overline{a_{ji}}$, $\forall\, i,\ j$, i.e. $A=A^\theta$.

Skew-hermitian Matrix A square matrix $A=[a_{ij}]$ is said to be a skew-hermitian matrix, if $a_{ij}=-\overline{a}_{ji}$, $\forall\, i,\ j$, i.e. $A^\theta=-A$.

PROPERTIES OF HERMITIAN AND SKEW-HERMITIAN MATRIX

(i) If A and B are hermitian matrices of same order, then $AB+BA$ is hermitian and $AB-BA$ is skew-hermitian matrix.

(ii) Every square matrix can be uniquely expressed as $A+iB$, where A and B are hermitian matrices.

(iii) Every square matrix can be uniquely expressed as the sum of a hermitian and a skew-hermitian matrices.

(iv) If A is a square matrix, then $A+A^\theta$ is hermitian and $A-A^\theta$ is skew-hermitian matrix.

(v) If A is a skew-hermitian matrix, then iA is hermitian and $\overline{A}$ is skew-hermitian matrix.

IDEMPOTENT MATRIX

A square matrix A is said to be idempotent, if $A^2=A$.

PROPERTIES OF IDEMPOTENT MATRIX

(i) If A and B are idempotent matrices, then AB is an idempotent matrix iff $AB=BA$.

(ii) If A and B are idempotent matrices, then $A+B$ is an idempotent iff $AB=BA=O$.

(iii) If A is an idempotent matrix and $A+B=I$, then B is an idempotent and $AB=BA=O$.

(iv) Diagonal $(1,1,1\ldots,1)$ is an idempotent matrix.

(v) If $AB=A$ and $BA=B$, then $A^2=A, B^2=B$.

INVOLUNTARY MATRIX

A square matrix A is said to be involuntary matrix, if $A^2=I$.

NILPOTENT MATRIX

A square matrix A is said to be nilpotent matrix, if there exists a positive integer m such that $A^m=O$. If m is the least positive integer such that $A^m=O$, then m is called the index of the nilpotent matrix A.

PERIODIC MATRIX

A matrix A will be called a periodic matrix iff $A^{k+1}=A$ for some positive integer k. If however, k is the least positive integer for which $A^{k+1}=A$, then k is said to be the period of A.

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ALGORITHM TO COMPUTE THE INVERSE OF A MATRIX USING ELEMENTARY TRANSFORMATIONS

(i) Using elementary row transformations

***Step* I** Write $A=IA$

***Step* II** Perform a sequence of elementary row operations successively on A of LHS and I of RHS, till we obtain the result $I=BA$

***Step* III** Write $A^{-1}=B$

(ii) Using elementary column transformations

***Step* I** Write $A=AI$

***Step* II** Perform a sequence of elementary columns operations successively on A of LHS and I of RHS, till we obtain the result $I=AB$

Step III Write $A^{-1}=B$

Note If we obtain all zeroes in one or more rows of the matrix A on LHS, then A^{-1} does not exist.

- If $AB=BA$, then the matrices A and B are called commutative matrices.
- If A is a square matrix of order n, then A^2 is defined as AA. In general, $A^m=A\cdot A\ldots A$ (m times), where m is any positive integer.
- If I is the unit matrix, then $I=I^2=I^3=\ldots=I^n$.
- Elements of main diagonal of a skew-symmetric matrix are all zero.
- If A is a square matrix, then
  (a) $A+A^T$ is symmetric matrix.
  (b) $A-A^T$ is skew-symmetric matrix.
- If A and B are two symmetric (or skew-symmetric) matrices of same order, then $A+B$ is symmetric (or skew-symmetric) matrix.
- Scalar multiple of symmetric (or skew-symmetric) matrix is also symmetric (or skew-symmetric) matrix.
- Product of two symmetric matrices A and B is symmetric matrix, if $AB=BA$.
- Every square matrix can be expressed uniquely as sum of symmetric and skew-symmetric matrices.
- The matrix $B^T AB$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric matrix.
- All positive integral powers of symmetric matrix are symmetric.
- All positive odd powers of skew-symmetric matrix is skew-symmetric though, even powers of skew-symmetric matrix is symmetric.
- If A and B are two symmetric matrices of the same order, then
  (a) $AB-BA$ is a skew-symmetric matrix.
  (b) $AB+BA$ is a symmetric matrix.
- Two matrices are said to be equivalent, if one is obtained from the other by elementary transformations. The symbol '$\approx$' is used for equivalence.
- (i) $(A^{-1})^{-1}=A$ (ii) $(A^T)^{-1}=(A^{-1})^T$
  (iii) $(AB)^{-1}=B^{-1}A^{-1}$

WBJEE
PRACTICE PAPER (2016)

Directions (Q. Nos. 1-50) *Each question has one correct option and carries* 1 *mark, for each wrong answer* 1/4 *mark will be deducted.*

1. The number of real roots of the equation $x^4 + x^2 + 2 = \log_{0.75}(2016)$ is

(a) 0 *(b)* 2
(c) infinite *(d)* 1

2. Maximum value of the determinant $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ is

(a) – 1 *(b)* 0 *(c)* 2 *(d)* 3

3. $\int_{\pi/2}^{3\pi/2} [2\sin x]\,dx$ is equal to ([·] denotes the greatest integer function)

(a) $-\pi$ *(b)* 0
(c) $-\frac{\pi}{2}$ *(d)* $\frac{\pi}{2}$

4. A natural number x is chosen at random from the first 100 natural numbers. The probability that $x + \frac{100}{x} > 50$ is

(a) $\frac{1}{10}$ *(b)* $\frac{11}{50}$
(c) $\frac{11}{20}$ *(d)* None of these

5. The curve represented by the differential equation $xdy - ydx = y\,dy$ intersects the Y-axis at $A(0, 1)$ and the line $y = e$ at (a, b), then the value of $a + b$ is

(a) 0 *(b)* 1 *(c)* 2 *(d)* – 1

6. Let $E = \{1, 2, 3, 4\}$ and $F = \{a, b\}$. Then, the number of onto functions from E to F is

(a) 14 *(b)* 16
(c) 12 *(d)* 32

7. The range of function $f(x) = {}^{7-x}P_{x-3}$ is

(a) {1, 2, 3, 4} *(b)* {0, 1, 2, 3, 4, 5}
(c) {0, 1, 2, 3} *(d)* {1, 2, 3, 4, 5, 6}

8. The minimum number of times a fair coin must be tossed, so that the probability of getting atleast one head is atleast 0.8, is

(a) 7 *(b)* 6
(c) 5 *(d)* 3

9. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, then the number of outcomes that B must have, so that A and B are independent, is

(a) 2, 4 or 8 *(b)* 3, 6 or 9
(c) 4 or 8 *(d)* 5 or 10

10. The value of $[\mathbf{a} - \mathbf{b} \;\; \mathbf{b} - \mathbf{c} \;\; \mathbf{c} - \mathbf{a}]$ is

(a) 0 *(b)* 1
(c) 2 *(d)* None of these

11. If two sides of a triangle are the roots of the equation $4x^2 - (2\sqrt{6})x + 1 = 0$ and the included angle is 60°, then the third side is

(a) $\sqrt{3}$ *(b)* $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{\sqrt{3}}$ *(d)* $2\sqrt{3}$

12. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$, then A is a nilpotent matrix of index

(a) 2 *(b)* 3
(c) 4 *(d)* 5

13. The sum S_n to n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$ is equal to

(a) $2^n - n - 1$ *(b)* $1 - 2^{-n}$
(c) $2^{-n} + n - 1$ *(d)* $2^n - 1$

14. If n is a positive integer, then $5^{2n+2} - 24n - 25$ is divisible by

(a) 576 *(b)* 575
(c) 675 *(d)* 674

15. Let A and B be two sets, then $(A \cup B)' \cup (A' \cap B)$ is equal to

(a) A' (b) A

(c) B' (d) None of these

16. Which of the following functions is inverse of itself?

(a) $f(x) = \frac{1-x}{1+x}$ (b) $f(x) = 3^{\log x}$

(c) $f(x) = 3^{x(x+1)}$ (d) None of these

17. The value of

$({}^7C_0 + {}^7C_1) + ({}^7C_1 + {}^7C_2) + \ldots + ({}^7C_6 + {}^7C_7)$ is

(a) $2^8 - 1$ (b) $2^8 + 1$

(c) 2^8 (d) $2^8 - 2$

18. The average of four digit numbers that can be formed using each of the digits 3, 5, 7 and 9 exactly once in each number, is

(a) 4444 (b) 5555

(c) 6666 (d) 7777

19. Let $f(x) = 1 + 2x^2 + 2^2x^4 + \ldots + 2^{10}x^{20}$. Then, $f(x)$ has

(a) more than one minimum (b) exactly one minimum

(c) atleast one maximum (d) None of these

20. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $[0, \pi]$

21. If $f(x)$ is differentiable and $\int_0^{t^2} x\, f(x)\, dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

(a) $\frac{2}{5}$ (b) $-\frac{5}{2}$

(c) 1 (d) $\frac{5}{2}$

22. Let l_1 and l_2 be two lines intersecting at P. If A_1, B_1, C_1 are points on l_1 and A_2, B_2, C_2, D_2, E_2 are points on l_2 and if none of these coincides with P, then the number of triangles formed by these eight points is

(a) 56 (b) 55

(c) 46 (d) 45

23. The quadratic equation $x^2 + 15|x| + 14 = 0$ has

(a) only positive solutions

(b) only negative solutions

(c) no solution

(d) both positive and negative solutions

24. The quadratic equation whose roots are $\sin^2 18°$ and $\cos^2 36°$, is

(a) $16x^2 - 12x + 1 = 0$ (b) $16x^2 + 12x + 1 = 0$

(c) $16x^2 - 12x - 1 = 0$ (d) $16x^2 + 10x + 1 = 0$

25. If for a plane, the intercepts on the coordinate axes are 8, 4, 4, then the length of the perpendicular from the origin onto the plane is

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$

(c) 3 (d) $\frac{4}{3}$

26. If ω is imaginary root of unity, then the value of $\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$ is

(a) $a^3 + b^3 + c^3$ (b) $a^2b - b^2c$

(c) 0 (d) $a^3 + b^3 + c^3 - 3abc$

27. Consider the system of equations in x, y, z as

$$x \sin 3\theta - y + z = 0,$$
$$x \cos 2\theta + 4y + 3z = 0$$
and $$2x + 7y + 7z = 0$$

If this system has a non-zero solution, then for integer n, values of θ are given by

(a) $\pi\left[n + \frac{(-1)^n}{3}\right]$ (b) $\pi\left[n + \frac{(-1)^n}{4}\right]$

(c) $\pi\left[n + \frac{(-1)^n}{6}\right]$ (d) $\frac{n\pi}{2}$

28. The length of the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is

(a) $\frac{9}{2}$ (b) $2\sqrt{2}$

(c) $3\sqrt{2}$ (d) $\frac{3}{2}$

29. The value of

$\sin\left(\sin^{-1}\frac{1}{3} + \sec^{-1}3\right) + \cos\left(\tan^{-1}\frac{1}{2} + \tan^{-1}2\right)$ is

(a) 1 (b) 2

(c) 3 (d) 4

30. Let $f(x) = \begin{cases} 2e, & \text{if } x \le 1 \\ \log(x-1), & \text{if } x > 1 \end{cases}$. The equation of the normal to $y = f(x)$ at the point (3, log 2) is

(a) $y - 2x = 6 + \log 2$

(b) $y + 2x = 6 + \log 2$

(c) $y + 2x = 6 - \log 2$

(d) $y + 2x = -6 + \log 2$

31. Let z_1 and z_2 be the roots of the equation $z^2 + pz + q = 0$, where p, q are real. The points represented by z_1, z_2 and the origin form an equilateral triangle, if

(a) $p^2 = 3q$ (b) $p^2 > 3q$

(c) $p^2 < 3q$ (d) $p^2 = 2q$

32. $\int (\sqrt[3]{x})\,(\sqrt[5]{1+\sqrt[3]{x^4}})\,dx$ is equal to

(a) $(1+x^{3/4})^{6/5}+C$ (b) $(1+x^{4/3})^{6/5}+C$

(c) $\frac{5}{8}(1+x^{4/3})^{6/5}+C$ (d) $\frac{1}{6}(1+x^{4/3})^{6}+C$

33. The equation of the sides of a triangle are $x-3y=0$, $4x+3y=5$ and $3x+y=0$. The line $3x-4y=0$ passes through

(a) the incentre (b) the centroid

(c) the orthocentre (d) the circumcentre

34. The number of common tangents to two circles $x^2+y^2=4$ and $x^2+y^2-8x+12=0$ is

(a) 1 (b) 3

(c) 5 (d) 4

35. If a point P moves such that its distances from the point $A(1,1)$ and the line $x+y+2=0$ are equal, then the locus of P is

(a) a straight line (b) a pair of straight line

(c) a parabola (d) an ellipse

36. If the derivative of the function $f(x)$ is everywhere continuous and is given by

$$f(x)=\begin{cases} bx^2+ax+4; & x\geq -1 \\ ax^2+b; & x<-1 \end{cases}, \text{ then}$$

(a) $a=2, b=-3$ (b) $a=3, b=2$

(c) $a=-2, b=-3$ (d) $a=-3, b=-2$

37. $\lim\limits_{n\to\infty}\dfrac{3\cdot 2^{n+1}-4\cdot 5^{n+1}}{5\cdot 2^n+7\cdot 5^n}$ is equal to

(a) $\frac{3}{5}$ (b) $-\frac{4}{7}$

(c) $-\frac{20}{7}$ (d) 0

38. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}e^{r/n}$ is equal to

(a) e (b) $e-1$

(c) $1-e$ (d) $e+1$

39. The area enclosed within the curve $|x|+|y|=1$, is

(a) 1 sq unit (b) $2\sqrt{2}$ sq units

(c) $\sqrt{2}$ sq units (d) 2 sq units

40. The equation of line of intersection of the planes $x+2y+z=3$ and $6x+8y+3z=13$ can be written as

(a) $\frac{x-2}{2}=\frac{y+1}{-3}=\frac{z-3}{4}$

(b) $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-3}{4}$

(c) $\frac{x+2}{2}=\frac{y-1}{-3}=\frac{z-3}{4}$

(d) $\frac{x+2}{2}=\frac{y+2}{3}=\frac{z-3}{4}$

41. If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are three non-coplanar vectors, then the vector equation $\mathbf{r}=(1-p-q)\,\mathbf{a}+p\,\mathbf{b}+q\,\mathbf{c}$ represents a

(a) standard line

(b) plane

(c) plane passing through origin

(d) None of the above

42. $(\mathbf{a}\times\mathbf{b})^2+(\mathbf{a}\cdot\mathbf{b})^2$ is equal to

(a) $\mathbf{a}^2\mathbf{b}^2$ (b) $\mathbf{a}^2+\mathbf{b}^2$

(c) 1 (d) $2\mathbf{a}\cdot\mathbf{b}$

43. The value of $\cot 70^\circ+4\cos 70^\circ$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\frac{1}{2}$

44. The function $f(x)=x(x+3)e^{-\left(\frac{1}{2}\right)x}$ satisfies the conditions of Rolle's theorem in $[-3,0]$. The value of c is

(a) 0 (b) -1

(c) -2 (d) -3

45. The function $f(x)=\sin^4 x+\cos^4 x$ increases, if

(a) $0<x<\frac{\pi}{8}$ (b) $\frac{\pi}{4}<x<\frac{3\pi}{8}$

(c) $\frac{3\pi}{8}<x<\frac{5\pi}{8}$ (d) $\frac{5\pi}{8}<x<\frac{3\pi}{4}$

46. Solution of $\dfrac{dy}{dx}=\dfrac{x\log x^2+x}{\sin y+y\cos y}$ is

(a) $y\sin y=x^2\log x+C$

(b) $y\sin y=x^2+C$

(c) $y\sin y=x^2+\log x$

(d) $y\sin y=x\log x+C$

47. The second order derivative of $a\sin^3 t$ with respect to $a\cos^3 t$ at $t=\frac{\pi}{4}$ is

(a) 2 (b) $\frac{1}{12a}$

(c) $\frac{4\sqrt{2}}{3a}$ (d) $\frac{3a}{4\sqrt{2}}$

48. If a point $P(4,3)$ is shifted by a distance $\sqrt{2}$ units parallel to the line $y=x$, then coordinates of P in new position are

(a) $(5,4)$ (b) $(5+\sqrt{2},4+\sqrt{2})$

(c) $(5-\sqrt{2},4-\sqrt{2})$ (d) None of these

49. The locus of a point which moves such that the sum of the squares of its distance from three vertices of the triangle is constant is a/an

(a) circle (b) straight line

(c) ellipse (d) None of these

50. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$, then x is equal to

(a) 9 (b) – 9
(c) 0 (d) – 1

Directions (Q. Nos. 51-65) *Each question has one correct option and carries* 2 *marks, for each wrong answer* 1/2 *mark will be deducted.*

51. The value of $f(0)$, so that the function $f(x) = \dfrac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$ becomes continuous, is

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$
(c) 2 (d) $\frac{1}{3}$

52. If $f\left(\dfrac{3x-4}{3x+4}\right) = x + 2$, then $\int f(x)\,dx$ is equal to

(a) $e^{x+2} \log \left| \dfrac{3x-4}{3x+4} \right| + C$

(b) $-\dfrac{8}{3} \log |1 - x| + \dfrac{2}{3} x + C$

(c) $\dfrac{8}{3} \log |1 - x| + \dfrac{x}{3} + C$

(d) $e^{[(3x-4)/(3x+4)]} - \dfrac{x^2}{2} - 2x + C$

53. The number of roots of the equation $x + 2 \tan x = \dfrac{\pi}{2}$ in the interval $[0, 2\pi]$ is

(a) 1 (b) 2
(c) 3 (d) infinite

54. The number of roots of the equation $2^x + 2^{x-1} + 2^{x-2} = 7^x + 7^{x-1} + 7^{x-2}$ is

(a) 4 (b) 2
(c) 1 (d) 0

55. The sum of the divisors of $2^5 \cdot 3^7 \cdot 5^3 \cdot 7^2$ is

(a) $2^6 \cdot 3^8 \cdot 5^4 \cdot 7^3$

(b) $2^6 \cdot 3^8 \cdot 5^4 \cdot 7^3 - 2 \cdot 3 \cdot 5 \cdot 7$

(c) $2^6 \cdot 3^8 \cdot 5^4 \cdot 7^3 - 1$

(d) None of the above

56. A letter is known to have come from either TATANAGAR or CALCUTTA. On the envelope, just two consecutive letters, TA, are visible. The probability that the letter has come from CALCUTTA is

(a) $\frac{4}{11}$ (b) $\frac{1}{3}$
(c) $\frac{5}{12}$ (d) $\frac{1}{7}$

57. The value of k for which the points $A(1, 0, 3)$, $B(-1, 3, 4)$, $C(1, 2, 1)$ and $D(k, 2, 5)$ are coplanar, is

(a) 1 (b) 2
(c) 0 (d) – 1

58. The perimeter of a triangle is 6 times the arithmetic mean of the sines of its angles. If the side a is 1, then A is equal to

(a) 30° (b) 60°
(c) 90° (d) 120°

59. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 is equal to

(a) 4 B (b) 128 B
(c) – 128 B (d) – 64 B

60. The period of the function $f(x) = \text{cosec}^2\, 3x + \cot 4x$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) π

61. If a, b, c are three positive real numbers, then least value of the expression $(a + b + c)\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$ is

(a) 3 (b) $3\sqrt{3}$
(c) 6 (d) 9

62. A variable triangle is inscribed in a circle of radius R. If the rate of change of a side is R times the rate of change of the opposite angle, then that angle is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

63. The value of $\left\{\dfrac{5^{99}}{13}\right\}$, (where $\{\cdot\}$ denotes the fractional part function) is

(a) $\frac{19}{28}$ (b) $\frac{8}{13}$
(c) $\frac{1}{13}$ (d) $\frac{3}{28}$

64. Let $f(x) = \text{minimum } (x + 1, \sqrt{1-x})$ for all $x \le 1$. Then, the area bounded by $y = f(x)$ and the X-axis is

(a) $\frac{7}{3}$ sq units (b) $\frac{1}{6}$ sq unit
(c) $\frac{11}{6}$ sq units (d) $\frac{7}{6}$ sq unit

65. Orthogonal trajectories of family of the curves $x^{2/3} + y^{2/3} = a^{2/3}$, where a is any arbitrary constant, is

(a) $x^{2/3} - y^{2/3} = C$ (b) $x^{4/3} - y^{4/3} = C$
(c) $x^{4/3} + y^{4/3} = C$ (d) $x^{1/3} - y^{1/3} = C$

Directions (Q. Nos. 66-75) *Each question has one or more correct option(s), choosing which will fetch 2 marks and choice of any wrong option(s) will fetch zero mark. However, when all correct options are not marked and also no incorrect options is marked, then marks awarded*

$$= 2 \times \left[\frac{\text{(number of correct option marked)}}{\text{(total number of correct options)}}\right]$$

66. Two fair dice are thrown. The probability that difference between the numbers is

(a) two, is $\frac{2}{9}$

(b) three, is $\frac{1}{6}$

(c) atleast two, is $\frac{1}{2}$

(d) atmost one is $\frac{4}{9}$

67. The equation of tangent parallel to $y = x$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

(a) $x - y + 1 = 0$

(b) $x - y - 2 = 0$

(c) $x + y - 1 = 0$

(d) $x - y - 1 = 0$

68. If $A = \lim\limits_{x \to -2} \frac{\tan \pi x}{x + 2} + \lim\limits_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$, then

(a) $A > 3$

(b) $A > 4$

(c) $A < 4$

(d) A is a transcedental number

69. The function $f(x) = \log (x - 2)^2 - x^2 + 4x + 1$ increases on the interval

(a) (1, 2) *(b)* (2, 3)

(c) $\left(\frac{5}{2}, 3\right)$ *(d)* (2, 4)

70. Let $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ x - 4, & x \geq 3 \end{cases}$

and $g(x) = \begin{cases} x - 3, & x < 4 \\ x^2 + 2x + 2, & x \geq 4 \end{cases}$

Then, which of the following is/are true?

(a) $(f + g)(3.5) = 0$

(b) $f(g(3)) = 0$

(c) $(fg)(2) = 1$

(d) $(f - g)(4) = 0$

71. If $f(x)$, $g(x)$ are twice differentiable functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9$, then

(a) $f(4) - g(4) = 10$

(b) $|f(x) - g(x)| < 2 \Rightarrow -2 < x < 0$

(c) $f(2) = g(2) \Rightarrow x = -1$

(d) $f(x) - g(x) = 2x$ has real root

72. If $\cos 5\theta = p \cos \theta + q \cos^3 \theta + r \cos^5 \theta + s$, then

(a) $p = 20$ *(b)* $q = -20$

(c) $r = 16$ *(d)* $s = 5$

73. The quadratic equation $x^2 - (m - 3)x + m = 0$ has

(a) real distinct roots if and only if $m \in (-\infty, 1) \cup (9, \infty)$

(b) both positive roots if and only if $m \in [9, \infty)$

(c) both negative roots if and only if $m \in (0, 1]$

(d) atleast one positive root if and only if $m \in (9, \infty)$

74. If $S = \sum\limits_{k=1}^{10} \left(\sin \frac{2\pi k}{11} - i \cos \frac{2\pi k}{11}\right)$, then

(a) $S + \bar{S} = 0$ *(b)* $S\bar{S} = 1$

(c) $\sqrt{S} = \pm \frac{1}{\sqrt{2}}(1 + i)$ *(d)* $S - \bar{S} = 0$

75. Number of ways in which 200 people can be divided in 100 couples is

(a) $\frac{(200)!}{2^{100}(100)!}$ *(b)* $1 \times 3 \times 5 \times \ldots \times 199$

(c) $\left(\frac{101}{2}\right)\left(\frac{102}{2}\right)\ldots\left(\frac{200}{2}\right)$ *(d)* $\frac{(200)!}{(100)!}$

Answers

1. (a)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (d)	**10.** (a)
11. (b)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (b)
21. (a)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (b)
31. (a)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (c)	**38.** (b)	**39.** (d)	**40.** (a)
41. (b)	**42.** (a)	**43.** (b)	**44.** (c)	**45.** (b)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (a)	**50.** (b)
51. (a)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (d)	**56.** (a)	**57.** (d)	**58.** (a)	**59.** (b)	**60.** (d)
61. (d)	**62.** (c)	**63.** (b)	**64.** (d)	**65.** (b)	**66.** (a,b,d)	**67.** (a,d)	**68.** (a,b,d)	**69.** (b,c)	**70.** (a,c)
71. (a,b,c)	**72.** (b,c)	**73.** (a,b,c)	**74.** (a,b,c)	**75.** (a,b,c)					

BRAIN Teasers

To boost up your ranks in JEE Main and Advanced

1. Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^2$.

If $(a-b)^2 + (p-q)^2 = 25$
$(b-c)^2 + (q-r)^2 = 36$
$(c-a)^2 + (r-p)^2 = 49$,

then find the value of $|B|$.

2. Find the number of solutions of the curves

$$y = \sin^{-1}(2x) + \tan^{-1}\left(\frac{1}{[2x]}\right) + 2$$

and $y = \cos^{-1}(2x+5) + 1$, where [ ] denotes the greatest integer function.

3. If $f(x) = \int 2^{\sin x \log_2 e}\left(\frac{5\cos x + \cos^3 x}{(2-\sin x)\sqrt{3+\cos^2 x}}\right)dx$

and $f(0) = 1$, then $f\left(\frac{\pi}{2}\right)$ is equal to

(a) $\frac{e}{\sqrt{3}}$ (b) $\sqrt{2}\,e$ (c) $\sqrt{3}\,e$ (d) $2e$

4. In ΔABC, the least value of $\frac{e^A}{A} + \frac{e^B}{B} + \frac{e^C}{C}$ is

(a) $\frac{\pi}{3}e^{\frac{\pi}{9}}$ (b) $\frac{\pi}{9}e^{\frac{\pi}{3}}$

(c) $\frac{9}{\pi}e^{\frac{\pi}{3}}$ (d) $\frac{3}{\pi}e^{\frac{\pi}{9}}$

5. A circle concentric to the ellipse $\frac{4x^2}{289} + \frac{y^2}{k^2} = 1$, where $k < \frac{17}{2}$, passes through foci S_1 and S_2 of the ellipse and cuts ellipse at P. If area of ΔPS_1S_2 is 35 sq units, then S_1S_2 is equal to

(a) 12 units (b) 13 units (c) $\sqrt{149}$ units (d) $\sqrt{49}$ units

6. Let $f(x) = \{-c^2 + (b-1)c - 2\}x + \int(\sin^2 x + \cos^4 x)\,dx$.

If $f(x)$ is an increasing function of $x, \forall\, x \in R$, then b lies in the interval

(a) $(1-\sqrt{5}, 1+\sqrt{5})$ (b) $[1-\sqrt{5}, 1+\sqrt{5}]$

(c) $(1+\sqrt{5}, 1-\sqrt{5})$ (d) None of these

7. A bag contains $n+1$ coins. It is known that one of these coins shows head on both sides, whereas the other coins are fair. Once coin is selected at random and tossed. If the probability that toss results in head is 7/12, then the value of n is

(a) 3 (b) 4

(c) 5 (d) None of these

8. If the 4th term in the expansion of $\left(px + \frac{1}{x}\right)^n$, $n \in N$ is $\frac{5}{2}$ and three normals to the parabola $y^2 = x$ are drawn through a point $(q, 0)$, then

(a) $q = p$ (b) $q > p$

(c) $q < p$ (d) $pq = 1$

9. Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be two unit vectors making angles θ with each other, then unit vectors along bisector of $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ is

(a) $\pm\frac{\hat{\mathbf{a}} + \hat{\mathbf{b}}}{2}$ (b) $\pm\frac{\hat{\mathbf{a}} + \hat{\mathbf{b}}}{2\cos\theta}$

(c) $\pm\frac{\hat{\mathbf{a}} + \hat{\mathbf{b}}}{2\cos\frac{\theta}{2}}$ (d) $\pm\frac{\hat{\mathbf{a}} + \hat{\mathbf{b}}}{|\hat{\mathbf{a}} + \hat{\mathbf{b}}|}$

10. The area of the region containing the points (x, y) satisfying $4 \le x^2 + y^2 \le 2(|x| + |y|)$ is

(a) 8 sq units
(b) 2 sq units
(c) 4π sq units
(d) 6 sq units

Answers *with* Explanation

1. ***(864)*** We know that, area of triangle having vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Clearly, $|A|$ is twice the area of triangle having vertices (a, p), (b, q) and (c, r) with sides 5, 6 and 7.

$$\therefore \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{9(9-5)(9-6)(9-7)}$$
$$\Rightarrow \quad \Delta^2 = 9 \times 4 \times 3 \times 2 = 216$$

We have, $B = A^2$

$$|B| = |A|^2 = (2\Delta)^2$$
$$\Rightarrow \quad |B| = 4\Delta^2$$
$$= 4 \times 216 = 864$$

2. ***(0)*** We have, $y = \sin^{-1}(2x) + \tan^{-1}\frac{1}{[2x]} + 2$ …(i)

Its domain is $-1 \le 2x \le 1$ and $[2x] \neq 0$.

Hence, domain is $[-\frac{1}{2}, 0) \cup \left\{\frac{1}{2}\right\}$ and $y = \cos^{-1}(2x + 5) + 1$

Its domain is $[-3, -2]$.

Since, there is no point of intersection. So, the number of solutions is zero.

3. ***(c)*** Clearly, $f(x) = \int e^{\sin x}\left[\frac{6 - \sin^2 x}{(2 - \sin x)\sqrt{4 - \sin^2 x}}\right]\cos x\, dx$

Put $\sin x = t$

$$\Rightarrow \quad \cos x\, dx = dt$$

Now, $$f(x) = \int e^t \frac{6 - t^2}{(2-t)\sqrt{4 - t^2}}\, dt$$

$$= \int e^t \left[\frac{2}{(2-t)\sqrt{4-t^2}} + \frac{4 - t^2}{(2-t)\sqrt{4-t^2}}\, dt\right]$$

$$= \int e^t \left[\frac{2}{(2-t)\sqrt{4-t^2}} + \sqrt{\frac{2+t}{2-t}}\right] dt$$

$$\therefore \quad f(x) = e^t\sqrt{\frac{2+t}{2-t}} + C \qquad [\because \int e^x[f(x) + f'(x)]dx = e^x f(x)]$$

$$\Rightarrow \quad f(x) = e^{\sin x}\sqrt{\frac{2 + \sin x}{2 - \sin x}} + C$$

$$\Rightarrow \quad f(0) = 1 = C + 1 \Rightarrow C = 0$$

Hence, $$f(x) = e^{\sin x}\sqrt{\frac{2 + \sin x}{2 - \sin x}} \Rightarrow f\left(\frac{\pi}{2}\right) = e\sqrt{3}$$

4. ***(c)*** We know that, AM ≥ GM

$$\therefore \quad \frac{e^A}{A} + \frac{e^B}{B} + \frac{e^C}{C} \ge 3\left(\frac{e^{A+B+C}}{ABC}\right)^{\frac{1}{3}} = 3\left(\frac{e^\pi}{ABC}\right)^{\frac{1}{3}}$$

Again, $A + B + C \ge 3(ABC)^{1/3}$ $[\because \text{AM} \ge \text{GM}]$

$$\frac{\pi}{3} \ge (ABC)^{\frac{1}{3}}$$

$$\frac{3}{\pi} \le \frac{1}{(ABC)^{\frac{1}{3}}}$$

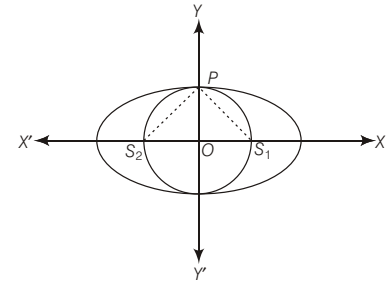
$$\Rightarrow \quad \frac{3}{\pi}(e^\pi)^{\frac{1}{3}} \le \left(\frac{e^\pi}{ABC}\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad \frac{e^A}{A} + \frac{e^B}{B} + \frac{e^C}{C} \ge 3\left(\frac{3}{\pi}\right)e^{\frac{\pi}{3}} = \frac{9}{\pi}e^{\frac{\pi}{3}}$$

Hence, least value is $\frac{9}{\pi}e^{\frac{\pi}{3}}$.

5. ***(c)*** Since, the circle is concentric to the ellipse and passes through the foci S_1 and S_2.

Hence, S_1S_2 is the diameter of the circle.



$\therefore$ The area of $\Delta PS_1S_2 = \frac{1}{2} \times S_1P \times S_2P$

We know that, $S_1P + S_2P = 2a = 17$

$$\text{Area} = \frac{1}{2}x(17 - x) = 35 \qquad [\text{let } S_1P = x]$$

$$\Rightarrow \quad x^2 - 17x + 70 = 0 \Rightarrow x = 7, 10$$

$$\therefore \quad S_1S_2 = \sqrt{PS_1^2 + PS_2^2}$$
$$= \sqrt{49 + 100} = \sqrt{149} \text{ units}$$

6. ***(a)*** We have, $f(x) = \{-c^2 + (b-1)c - 2\}x + \int(\sin^2 x + \cos^4 x)\, dx$

$$\Rightarrow \quad f'(x) = -c^2 + (b-1)c - 2 + \sin^2 x + \cos^4 x$$

Given, $f'(x) > 0, \forall\ x \in R$ and $\forall\ c \in R$

$$\Rightarrow \quad -c^2 + (b-1)c - 2 + \sin^2 x + \cos^4 x > 0$$
$$\Rightarrow \quad \sin^2 x + \cos^4 x > c^2 - (b-1)c + 2$$
$$\Rightarrow \quad \sin^2 x + (1 - \sin^2 x)^2 > c^2 - (b-1)c + 2$$
$$\Rightarrow \quad \sin^4 x - \sin^2 x + 1 > c^2 - (b-1)c + 2$$
$$\Rightarrow \quad \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} > c^2 - (b-1)c + 2$$
$$\Rightarrow \quad \frac{3}{4} > c^2 - (b-1)c + 2$$
$$\Rightarrow \quad 4c^2 - 4(b-1)c + 8 < 3$$
$$\Rightarrow \quad 4c^2 - 4(b-1)c + 5 < 0$$
$$\Rightarrow \quad [-4(b-1)]^2 - 4 \cdot 4 \cdot 5 < 0$$
$$\Rightarrow \quad (b-1)^2 - 5 < 0$$
$$\Rightarrow \quad -\sqrt{5} < (b-1) < \sqrt{5}$$
$$\Rightarrow \quad 1 - \sqrt{5} < b < 1 + \sqrt{5}$$

Hence, $b \in (1 - \sqrt{5}, 1 + \sqrt{5})$

7. ***(c)*** Let E_1 denotes the event 'a coin with two heads is selected' and E_2 denotes the event 'a fair coin is selected'. Let A be the event 'the toss results in heads '.

Then, $P(E_1) = \frac{1}{n+1}, P(E_2) = \frac{n}{n+1}, P\left(\frac{A}{E_1}\right) = 1$

and $P\left(\frac{A}{E_2}\right) = \frac{1}{2}$

$\therefore \quad P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)$

$\Rightarrow \quad \frac{7}{12} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$

$\Rightarrow \quad 12 + 6n = 7n + 7 \Rightarrow n = 5$

8. ***(b)*** Given, $\frac{5}{2} = {}^nC_3 (px)^{n-3}\left(\frac{1}{x}\right)^3 = {}^nC_3\, p^{n-3}\, x^{n-6}$ ...(i)

Since, LHS is independent of x.

$\therefore \quad n - 6 = 0 \Rightarrow n = 6$

$\frac{5}{2} = {}^6C_3\, p^3 = 20p^3 \Rightarrow p^3 = \left(\frac{1}{2}\right)^3 \Rightarrow p = \frac{1}{2}$

Given parabola is $y^2 = x$.

$\therefore \; 4a = 1 \Rightarrow a = \frac{1}{4}$

Since, three normals are drawn from $(q, 0)$.

$\therefore \quad q > 2a$ or $q > \frac{1}{2}$ or $q > p$

9. ***(c)*** In ΔABC,

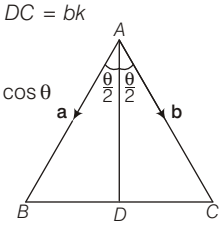
$\frac{BD}{DC} = \frac{a}{b} \Rightarrow BD = ak$ and $DC = bk$

$\Rightarrow \quad BC = (a + b)\, k$

Now, $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos\theta$

$\Rightarrow (a + b)^2 k^2 = a^2 + b^2 - 2ab\cos\theta$

$\because \quad k^2 = \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}$



In ΔABD, $\cos\frac{\theta}{2} = \frac{a^2 + AD^2 - a^2k^2}{2a \cdot AD}$

and in ΔADC, $\cos\frac{\theta}{2} = \frac{b^2 + (AD)^2 - b^2k^2}{2b \cdot AD}$

$\Rightarrow \quad AD^2 = ab(1 - k^2) = ab\left\{1 - \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}\right\}$

$= \frac{4a^2b^2\cos^2\frac{\theta}{2}}{(a+b)^2}$

$\therefore \quad AD = \frac{2ab\cos\frac{\theta}{2}}{a+b}$

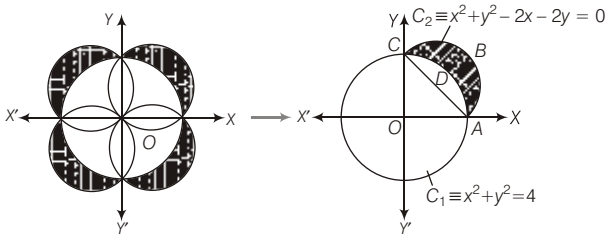
Now, $\mathbf{AD} = \pm\frac{\mathbf{ab} + \mathbf{ba}}{a+b} = \pm\frac{ab}{a+b}\left(\frac{\mathbf{a}}{a} + \frac{\mathbf{b}}{b}\right)$

$= \pm\frac{ab}{a+b}(\hat{\mathbf{a}} + \hat{\mathbf{b}})$

$\therefore \quad \hat{\mathbf{AD}} = \frac{\mathbf{AD}}{AD} = \pm\frac{\hat{\mathbf{a}} + \hat{\mathbf{b}}}{2\cos\frac{\theta}{2}}$

10. ***(a)*** The points in the required region satisfy

$4 \le x^2 + y^2 \le 2(|x| + |y|)$ ...(i)



Since, the curve (i) is symmetrical about both the axes , then the required area is 4 times the area of the region in the Ist quadrant.

Therefore, it is sufficient to sketch the region and to find the area in the Ist quadrant.

In the Ist quadrant, the curve has two curves

$x^2 + y^2 \ge 4$ and $x^2 + y^2 - 2x - 2y \le 0$

$\therefore$ Required area $= 4 \times$ Area of $ABCDA$

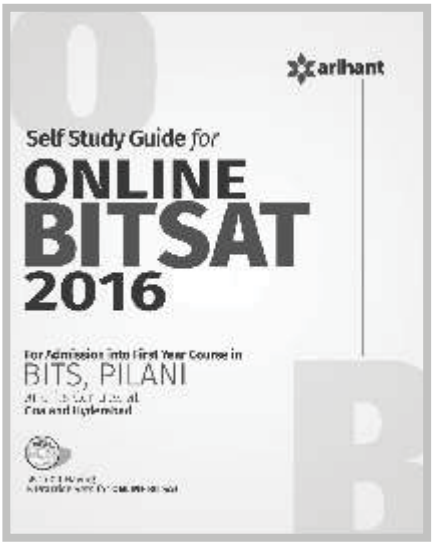
$= 4$ [(Area of semi-circle $ABCA$)–(Area of segment $ADCA$)]

$= 4$ [(Area of semi-circle $ABCA$) –

(Area of sector $OADCO$ – Area of ΔOAC)]

$= 4[\pi - (\pi - 2)] = 8$ sq units

Self Study Guide for
ONLINE
BITSAT 2016



C064 ₹775

TARGET BITSAT 2016

TEST Tuner

Comprehensive Simulator Test Series for BITSAT

BITSAT PREP UP

Mock Questions with Complete Solutions

Instructions

- This test consists of 45 questions and each question is allotted 3 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. If a, b, c are in GP and x, y are arithmetic mean of a, b and b, c respectively, then $\frac{1}{x}+\frac{1}{y}$ is equal to

(a) $\frac{2}{b}$ (b) $\frac{3}{b}$ (c) $\frac{b}{3}$ (d) $\frac{b}{2}$

2. If $\frac{2z_1}{3z_2}$ is purely imaginary, then $\left|\frac{z_1-z_2}{z_1+z_2}\right|$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) 1

3. If $e^{f(x)}=\frac{10+x}{10-x}$, $x\in(-10,10)$ and $f(x)=kf\left(\frac{200x}{100+x^2}\right)$, then k is equal to

(a) 0.5 (b) 0.6 (c) 0.7 (d) 0.8

4. The solution of the differential equation $xy^2\,dy-(x^3+y^3)\,dx=0$ is

(a) $y^3=3x^3+C$ (b) $y^3=3x^3\log(Cx)$

(c) $y^3=3x^3+\log(Cx)$ (d) $y^3+3x^3=\log(Cx)$

5. The number of positive integral solutions of the equation $\Delta=12$, where $\Delta=\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$ is

(a) 3 (b) 15 (c) 4 (d) $2^3\cdot 3^2$

6. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=\frac{1}{3}$. Then, value of λ is

(a) $-\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $-\frac{3}{5}$ (d) $\frac{5}{3}$

7. If $(2x^2-x-1)^5=a_0+a_1x+a_2x^2+\ldots+a_{10}x^{10}$, then $a_2+a_4+a_6+a_8+a_{10}$ is equal to

(a) 17 (b) 34

(c) 16 (d) 32

8. If the lengths of the sides of a right ΔABC right angled at C are in AP, then $5(\sin A+\sin B)$ is equal to

(a) 6 (b) 4

(c) 3 (d) 7

9. The value of α which satisfying $\int_{\pi/2}^{\alpha}\sin x\,dx=\sin 2\alpha$, $\alpha\in(0,2\pi)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$

(c) $\frac{7\pi}{6}$ (d) All of these

10. If the normal at $(ap^2, 2ap)$ on the parabola $y^2=4ax$ meet the parabola again at $(aq^2, 2aq)$, then

(a) $p^2+pq+2=0$ (b) $p^2-pq+2=0$

(c) $q^2+pq+2=0$ (d) $p^2+pq+1=0$

11. If $A=\begin{bmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{bmatrix}$, $i=\sqrt{-1}$ and $f(x)=x^2+2$, then $f(A)$ is equal to

(a) $\left(\frac{5-i\sqrt{3}}{2}\right)\begin{bmatrix}1&0\\0&1\end{bmatrix}$ (b) $\left(\frac{3-i\sqrt{3}}{2}\right)\begin{bmatrix}1&0\\0&1\end{bmatrix}$

(c) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ (d) $(2+i\sqrt{3})\begin{bmatrix}1&0\\0&1\end{bmatrix}$

12. A bag contains 8 red and 7 black balls. Two balls are drawn at random. The probability that both the balls are of the same colour, is

(a) $\frac{14}{15}$ (b) $\frac{11}{15}$

(c) $\frac{7}{15}$ (d) $\frac{4}{15}$

13. The period of the function $f(x) = \sin\left(\sin\frac{x}{5}\right)$ is

(a) 2π (b) $\frac{2\pi}{5}$

(c) 10π (d) 5π

14. If $f(x) = \begin{cases} \frac{1-\sin x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ \lambda, & x = \frac{\pi}{2} \end{cases}$, is continuous at $x = \frac{\pi}{2}$, then value of λ is

(a) –1 (b) 1

(c) 0 (d) 2

15. If x_1, x_2, x_3, x_4 are roots of the equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$ is equal to

(a) β (b) $\frac{\pi}{2} - \beta$

(c) $\pi - \beta$ (d) $-\beta$

16. The locus of the foot of the perpendicular drawn from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ on any tangent is given by $(x^2 + y^2)^2 = lx^2 + my^2$, where

(a) $l = a^2, m = b^2$ (b) $l = b^2, m = a^2$

(c) $l = m = a$ (d) $l = m = b$

17. The value of $\lim_{x \to 0} \frac{(1 - \cos 2x)\sin 5x}{x^2 \sin 3x}$ is

(a) 10/3 (b) 3/10

(c) 6/5 (d) 5/6

18. If A is the area formed by the positive X-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, then A is equal to

(a) $\sqrt{3}$ (b) $2\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) None of these

19. The sum of the coefficient in the expansion of $(1 + x - 3x^2)^{3148}$ is

(a) 8 (b) 7

(c) 1 (d) –1

20. If $I = \int \frac{dx}{x^{1/2} + x^{1/3}}$, then I is equal to

(a) $2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log(1 + x^{1/6}) + C$

(b) $3x^{1/3} - 6x^{1/6} - 6\log(1 + x^{1/6}) + C$

(c) $\sqrt{x} + 3x^{1/3} - 6x^{1/6} + 6\log(1 + x^{1/6}) + C$

(d) $3x^{1/3} - \sqrt{x} + 6x^{1/6} - 6\log(1 + x^{1/6}) + C$

21. The solution of the differential equation $\frac{dy}{dx} = \sin(x + y)\cdot\tan(x + y) - 1$ is

(a) $\operatorname{cosec}(x + y) + \tan(x + y) = x + C$

(b) $x + \operatorname{cosec}(x + y) = C$

(c) $x + \tan(x + y) = C$

(d) $x + \sec(x + y) = C$

22. A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not, is

(a) $\frac{1}{2}$ (b) $\frac{5}{9}$

(c) $\frac{4}{9}$ (d) $\frac{2}{3}$

23. The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$ is

(a) $\frac{10\pi}{3}$ (b) 0

(c) $\frac{\pi}{2}$ d) $\frac{5\pi}{3}$

24. A sample of 35 observations has the mean 80 and standard deviation 4. A second sample of 65 observations from the same population has mean 70 and standard deviation 3. The standard of the combined sample is

(a) 5.85 (b) 5.58

(c) 3.42 (d) None of these

25. The sum of the series $\log_9 3 + \log_{27} 3 - \log_{81} 3 + \log_{243} 3 + \ldots$ is

(a) $1 - \log_e 2$ (b) $1 + \log_e 2$ (c) $\log_e 2$ (d) $\log_e 3$

26. $ABCD$ is a square with vertices (0, 0), (1, 0), (1, 1) and (0, 1). P, Q, R and S are the points which divide AB, BC, CD and DA, respectively in the ratio 2 : 1. If the origin is shifted to the centre of the square $ABCD$ without rotation of axes, then the area of square $PQRS$ in the new system of coordinates is

(a) 2/9 sq unit (b) 4/9 sq unit

(c) 5/9 sq unit (d) None of these

27. The complete solution set of $\sin^{-1}(\sin 5) > x^2 - 4x$ is

(a) $|x - 2| < \sqrt{9 - 2\pi}$ (b) $|x - 2| > \sqrt{9 - 2\pi}$

(c) $|x| < \sqrt{9 - 2\pi}$ (d) $|x| > \sqrt{9 - 2\pi}$

28. If α, β, γ are the roots of $x^3 + 4x + 1 = 0$, then the equation whose roots are $\frac{\alpha^2}{\beta + \gamma}, \frac{\beta^2}{\gamma + \alpha}, \frac{\gamma^2}{\alpha + \beta}$, is

(a) $x^3 - 4x - 1 = 0$ (b) $x^3 - 4x + 1 = 0$

(c) $x^3 + 4x - 1 = 0$ (d) $x^3 + 4x + 1 = 0$

29. The area of the region bounded by the curves $y = x \log x$ and $y = 2x - 2x^2$ is

(a) $\frac{1}{12}$ sq unit (b) $\frac{3}{12}$ sq unit

(c) $\frac{7}{12}$ sq unit (d) None of these

30. If $\lim_{n\to\infty}\frac{1-(10)^n}{1+(10)^{n+1}}=\frac{-\alpha}{10}$, then the value of α is

(a) 0 (b) -1
(c) 1 (d) 2

31. The function $f(x)=2x^3-3x^2+90x+174$ is increasing in the interval

(a) $\frac{1}{2}<x<1$
(b) $\frac{1}{2}<x<2$
(c) $3<x<\frac{59}{4}$
(d) $-\infty<x<\infty$

32. The unit normal vector to the line joining $\hat{i}-\hat{j}$, $2\hat{i}+3\hat{j}$ and pointing toward the origin, is

(a) $\frac{4\hat{i}-\hat{j}}{\sqrt{17}}$ (b) $\frac{-4\hat{i}+\hat{j}}{\sqrt{17}}$
(c) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ (d) $\frac{-2\hat{i}+3\hat{j}}{\sqrt{13}}$

33. If $\frac{1}{e^{3x}}(e^x+e^{5x})=a_0+a_1x+a_2x^2+\ldots$,
then $2a_1+2^3a_3+2^5a_5+\ldots$ is equal to

(a) e (b) e^{-1}
(c) 1 (d) 0

34. The value of $\int_1^e 10^{\log_e x}\,dx$ is

(a) $10\log_e(10e)$ (b) $\frac{10e-1}{\log_e(10e)}$
(c) $\frac{10e}{(\log_e 10)e}$ (d) $\log_e(10e)$

35. In a bag there are six balls of unknown colours; three balls are drawn at random and found to be all black. The probability that no black ball is left in the bag is

(a) $\frac{1}{35}$ (b) $\frac{4}{35}$
(c) $\frac{6}{35}$ (d) None of these

36. The equation of the plane through the point $(2,-1,-3)$ and parallel to the line $\frac{x-1}{3}=\frac{y+2}{2}=\frac{z}{-4}$ and $\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2}$ is

(a) $8x+14y+13z+37=0$
(b) $8x+14y-13z+37=0$
(c) $8x-14y+13z+37=0$
(d) $8x+14y+13z-37=0$

37. The equation of straight line through the intersection of the lines $x-2y=1$ and $x+3y=2$ and parallel to $3x+4y=0$, is

(a) $3x+4y+5=0$
(b) $3x+4y-10=0$
(c) $3x+4y-5=0$
(d) $3x+4y+6=0$

38. The standard deviation of the first $2n+1$ natural numbers is

(a) $\sqrt{\frac{n(n+1)}{2}}$ (b) $\sqrt{\frac{n(n+1)(2n+1)}{3}}$
(c) $\sqrt{\frac{n(n+1)}{3}}$ (d) $\sqrt{\frac{n(n-1)}{2}}$

39. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is

(a) 2^{mn} (b) $2^{mn}-1$
(c) $2mn$ (d) m^n

40. $(p\wedge\sim q)\wedge(\sim p\vee q)$ is

(a) a contradiction (b) a tautology
(c) Either (a) or (b) (d) Neither (a) nor (b)

41. A firm makes pants and shirts. A shirt takes 2h on machine and 3h of man labour while a pant takes 3h on machine and 2 h of man labour. In a week there are 70 h of machine and 75 h of man labour available. If the firm decide to make x shirts and y pants per week, then for this the linear constraints are

(a) $x\geq 0,\ y\geq 0, 2x+3y\geq 70, 3x+2y\geq 75$
(b) $x\geq 0,\ y\geq 0, 2x+3y\leq 70, 3x+2y\geq 75$
(c) $x\geq 0,\ y\geq 0, 2x+3y\geq 70, 3x+2y\leq 75$
(d) $x\geq 0,\ y\geq 0, 2x+3y\leq 70, 3x+2y\leq 75$

42. Family of curves $y=e^x(A\cos x+B\sin x)$ represents the differential equation

(a) $\frac{d^2y}{dx^2}=2\frac{dy}{dx}-y$
(b) $\frac{d^2y}{dx^2}=2\frac{dy}{dx}-2y$
(c) $\frac{d^2y}{dx^2}=\frac{dy}{dx}-2y$
(d) $\frac{d^2y}{dx^2}=2\frac{dy}{dx}+y$

43. If $\sin A, \sin B, \cos A$ are in GP then the Roots of $x^2+2x\cot B+1=0$ are always

(a) real (b) imaginary
(c) greater than 1 (d) equal

44. If the relation $R: A\to B$, where $A=\{1,2,3,4\}$ and $B=\{1,3,5\}$, is defined by $R=\{(x,y): x<y, x\in A, y\in B\}$, then RoR^{-1} is

(a) $\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$
(b) $\{(3,1),(5,1),(5,2),(5,3)(5,4)\}$
(c) $\{(3,3),(3,5),(5,3),(5,5)\}$
(d) None of the above

45. Three straight lines l_1, l_2 and l_3 are parallel and lie in the same plane. Five points are taken on each of l_1, l_2 and l_3. The maximum number of triangles which can be obtained with vertices at these points, is

(a) 425 (b) 405
(c) 415 (d) 505

Answers *with* Explanation

1. **(a)** Consider, $\frac{1}{x}+\frac{1}{y}=\frac{2}{a+b}+\frac{2}{b+c}=\frac{2(2b+a+c)}{ab+b^2+ac+bc}$

$=\frac{2(2b+a+c)}{ab+2b^2+bc}$ $[\because b^2=ac]$

$=\frac{2(2b+a+c)}{b(2b+a+c)}=\frac{2}{b}$

2. **(d)** Let $\frac{2z_1}{3z_2}=ik \Rightarrow \frac{z_1}{z_2}=\frac{3ik}{2}$

$\Rightarrow \frac{z_1+z_2}{z_1-z_2}=\frac{3ik+2}{3ik-2} \Rightarrow \frac{z_1-z_2}{z_1+z_2}=\frac{3ik-2}{3ik+2}$

$\Rightarrow \left|\frac{z_1-z_2}{z_1+z_2}\right|=\left|\frac{3ik-2}{3ik+2}\right|=1$

3. **(a)** We have, $e^{f(x)}=\frac{10+x}{10-x}$, $x\in(-10,10)$

$\Rightarrow f(x)=\log\left(\frac{10+x}{10-x}\right)$

$\Rightarrow f\left(\frac{200x}{100+x^2}\right)=\log\left(\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right)$

$=\log\left(\frac{10(10+x)^2}{10(10-x)^2}\right)$

$=2\log\left(\frac{10+x}{10-x}\right)=2f(x)$

$\therefore f(x)=\frac{1}{2}f\left(\frac{200x}{100+x^2}\right)$

Hence, $k=\frac{1}{2}=0.5$

4. **(b)** Given, differential equation can be rewritten as $\frac{dy}{dx}=\frac{x^3+y^3}{xy^2}$

$\therefore x\frac{dv}{dx}+v=\frac{x^3+v^3x^3}{x^3v^2}=\frac{1+v^3}{v^2}$

[put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$]

$\Rightarrow \int v^2dv=\int\frac{dx}{x} \Rightarrow \frac{1}{3}\left(\frac{y}{x}\right)^3=\log x+\log C$

$\Rightarrow y^3=3x^3\log Cx$

5. **(a)** We have, $\Delta=\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$

On applying $C_1\to C_1+C_2+C_3$, we get

$\Delta=2\begin{vmatrix} y+z & z & y \\ z+x & z+x & x \\ x+y & x & x+y \end{vmatrix}$

On applying $C_2\to C_2-C_1$ and $C_3\to C_3-C_1$, we get

$\Delta=2\begin{vmatrix} y+z & -y & -z \\ z+x & 0 & -z \\ x+y & -y & 0 \end{vmatrix}$

On applying $C_1\to C_1+C_2+C_3$, we get

$\Delta=2\begin{vmatrix} 0 & -y & -z \\ x & 0 & -z \\ x & -y & 0 \end{vmatrix}=2\,xyz\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$=2xyz[-1(-1)+1(1)]=4xyz$

$\because \quad \Delta=12 \Rightarrow 4xyz=12\Rightarrow xyz=3$

Now, the number 3 can be assigned to any of x, y, z. Therefore, the number of positive integral solutions of $\Delta=12$ is 3.

6. **(d)** We know, $\sin\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

where, a_1, b_1, c_1 are DR's of line and a_2, b_2, c_2 are DR's of normal to the plane.

Here, we have

$a_1=1, b_1=2, c_1=2$

and $a_2=2, b_2=-1, c_2=\sqrt{\lambda}$

$\therefore \sin\theta=\left|\frac{1\cdot2+2\cdot(-1)+2\cdot\sqrt{\lambda}}{\sqrt{1^2+2^2+2^2}\sqrt{2^2+(-1)^2+(\sqrt{\lambda})^2}}\right|=\left|\frac{2\sqrt{\lambda}}{3\cdot\sqrt{5+\lambda}}\right|$

$\Rightarrow \frac{1}{3}=\left|\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}\right| \Rightarrow \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}=\frac{1}{3}$

$\Rightarrow 2\sqrt{\lambda}=\sqrt{5+\lambda}$

On squaring both sides, we get

$4\lambda=5+\lambda \Rightarrow \lambda=\frac{5}{3}$

7. **(a)** We have, $(2x^2-x-1)^5=a_0+a_1x+a_2x^2+\ldots+a_{10}x^{10}$

On putting $x=0$, we get

$(-1)^5=a_0\Rightarrow a_0=-1$ …(i)

On putting $x=1$, we get

$0^5=a_0+a_1+a_2+\ldots+a_{10}$

$\Rightarrow a_0+a_1+a_2+\ldots+a_{10}=0$ …(ii)

On putting $x=-1$, we get

$2^5=a_0-a_1+a_2-a_3+\ldots+a_{10}$

$\Rightarrow a_0-a_1+a_2-a_3+\ldots+a_{10}=32$ …(iii)

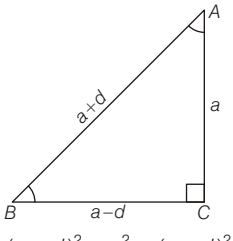
On adding Eqs. (ii) and (iii), we get

$2a_0+2a_2+\ldots+2a_{10}=32$

$\Rightarrow a_0+a_2+\ldots+a_{10}=16$

$\Rightarrow a_2+a_4+\ldots+a_{10}=17$

8. **(d)** Let the sides be $a-d, a, a+d$. Then, we have,



$(a+d)^2=a^2+(a-d)^2$

$\Rightarrow \quad a^2 + d^2 + 2ad = a^2 + a^2 + d^2 - 2ad$

$\Rightarrow \quad a^2 = 4ad$

$\Rightarrow \quad a = 4d \quad [\because a \neq 0]$

Also, from ΔABC, we have

$$\frac{a-d}{\sin A} = \frac{a}{\sin B} = \frac{a+d}{\sin 90^\circ}$$

$$\Rightarrow \quad \frac{a-d}{\sin A} = \frac{a}{\sin B} = \frac{a+d}{1}$$

$$\Rightarrow \quad \sin A = \frac{a-d}{a+d} = \frac{4d-d}{4d+d} = \frac{3}{5}$$

and $\quad \sin B = \frac{a}{a+d} = \frac{4d}{5d} = \frac{4}{5}$

Hence, $\quad 5(\sin A + \sin B) = 7$

9. ***(d)*** We have, $\int_{\pi/2}^{\alpha} \sin x\, dx = \sin 2\alpha, \alpha \in (0, 2\pi)$

$\Rightarrow \quad -(\cos x)_{\pi/2}^{\alpha} = \sin 2\alpha$

$\Rightarrow \quad -\cos\alpha = \sin 2\alpha$

$\Rightarrow \quad -\cos\alpha = 2\sin\alpha\cos\alpha$

$\Rightarrow \quad 2\sin\alpha\cos\alpha + \cos\alpha = 0$

$\Rightarrow \quad \cos\alpha(2\sin\alpha + 1) = 0$

$\Rightarrow \quad \cos\alpha = 0 \text{ or } \sin\alpha = -\frac{1}{2} = \sin\frac{7\pi}{6} \text{ or } \sin\frac{11\pi}{6}$

$\therefore \quad \alpha = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}$

10. ***(a)*** We know that, if the normal drawn at a point $P(at_1^2, 2at_1)$ to the parabola $y^2 = 4ax$ meets again the parabola at $Q(at_2^2, 2at_2)$, then

$$t_2 = -t_1 - \frac{2}{t_1}$$

Here, $\quad t_1 = p \quad \text{and} \quad t_2 = q$

$\Rightarrow \quad q = -p - \frac{2}{p} \Rightarrow q = \frac{-p^2 - 2}{p}$

$\Rightarrow \quad p^2 + pq + 2 = 0$

11. ***(d)*** We have, $A = \begin{bmatrix} \frac{\omega}{i} & \frac{\omega^2}{i} \\ -\frac{\omega^2}{i} & -\frac{\omega}{i} \end{bmatrix} = \frac{\omega}{i}\begin{bmatrix} 1 & \omega \\ -\omega & -1 \end{bmatrix}$

Now, $\quad A^2 = -\omega^2\begin{bmatrix} 1-\omega^2 & 0 \\ 0 & 1-\omega^2 \end{bmatrix}$

$$= \begin{bmatrix} -\omega^2 + \omega & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix}$$

and $\quad f(A) = A^2 + 2$

$$= \begin{bmatrix} -\omega^2 + \omega & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= (3 + 2\omega)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (2 + i\sqrt{3})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

12. ***(c)*** Required probability

= P(Either the balls are red or the balls are black)

$$= \frac{{}^8C_2}{{}^{15}C_2} + \frac{{}^7C_2}{{}^{15}C_2} = \frac{49}{105} = \frac{7}{15}$$

13. ***(c)*** Consider, $f(x + 10\pi) = \sin\left\{\sin\left(\frac{x + 10\pi}{5}\right)\right\}$

$$f(x + 10\pi) = \sin\left\{\sin\left(\frac{x}{5} + 2\pi\right)\right\}$$

$$f(x + 10\pi) = \sin\left\{\sin\left(\frac{x}{5}\right)\right\} = f(x)$$

$\therefore$ Period of $f(x)$ is 10π.

14. ***(c)*** Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$, therefore

$$\lim_{x \to \pi/2} f(x) = f\left(\frac{\pi}{2}\right) \text{ or } \lambda = \lim_{x \to \pi/2} \frac{1 - \sin x}{\pi - 2x}$$

Applying L-Hospital's rule, we get

$$\lambda = \lim_{x \to \pi/2} \frac{-\cos x}{-2} \Rightarrow \lambda = 0$$

15. ***(b)*** Clearly, $\Sigma x_1 = \sin 2\beta, \Sigma x_1x_2 = \cos 2\beta$

$$\Sigma x_1x_2x_3 = \cos\beta \quad \text{and} \quad x_1x_2x_3x_4 = -\sin\beta$$

Now, consider, $\tan^{-1}x_1 + \tan^{-1}x_2 + \tan^{-1}x_3 + \tan^{-1}x_4$

$$= \tan^{-1}\left(\frac{\Sigma x_1 - \Sigma x_1x_2x_3}{1 - \Sigma x_1x_2 + x_1x_2x_3x_4}\right)$$

$$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \beta\right)\right] = \frac{\pi}{2} - \beta$$

16. ***(a)*** Equation of tangent of slope m to the ellipse are

$$y = mx \pm \sqrt{a^2m^2 + b^2}.$$

$$y - mx = \pm\sqrt{a^2m^2 + b^2} \quad \ldots(i)$$

Now, equation of perpendicular line is $my + x = \lambda$

$\because$ It passes through the centre (0, 0).

$\therefore \quad my + x = 0 \quad \ldots(ii)$

On squaring and adding Eqs. (i) and (ii), we get

$$y^2 + m^2x^2 + m^2y^2 + x^2 = a^2m^2 + b^2$$

$$\Rightarrow \quad (x^2 + y^2)^2 = a^2x^2 + b^2y^2 \quad \left(\because m = -\frac{x}{y}\right)$$

$$\Rightarrow \quad l = a^2, m = b^2$$

17. ***(a)*** Consider, $\lim_{x \to 0} \frac{(1 - \cos 2x)\sin 5x}{x^2 \sin 3x}$

$$= \lim_{x \to 0} \frac{2\sin^2 x \sin 5x}{x^2 \sin 3x}$$

$$= \lim_{x \to 0}\left(\frac{2\sin^2 x}{x^2}\right)\left(\frac{\frac{\sin 5x}{x}}{\frac{\sin 3x}{x}}\right) = 2 \times \frac{5}{3} = \frac{10}{3}$$

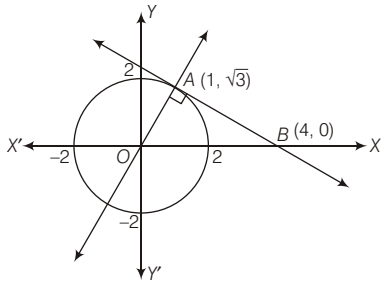
18. ***(b)*** Equation of normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

$$\frac{x}{1} = \frac{y}{\sqrt{3}} \Rightarrow \sqrt{3}x - y = 0$$

and equation of tangent is $x \cdot 1 + y \cdot \sqrt{3} = 4$

$$\Rightarrow \quad x + \sqrt{3}y = 4$$

Clearly, normal intersect X-axis at (0, 0) and tangent intersect X-axis at (4, 0).



Hence, the required area = Area of $\Delta AOB = \frac{1}{2}\cdot 2\cdot 2\sqrt{3} = 2\sqrt{3}$

19. **(c)** On substituting $x = 1$ in $(1 + x - 3x^2)^{3148}$, we get the sum of coefficient $= (1 + 1 - 3)^{3148} = (-1)^{3148} = 1$

20. **(a)** We have, $I = \int \frac{dx}{x^{1/2} + x^{1/3}} = \int \frac{dx}{x^{1/3}(1 + x^{1/6})}$

Now, put $x^{1/6} = t$ or $x = t^6 \Rightarrow dx = 6t^5 dt$

$$\therefore \quad I = 6\int \frac{t^5\,dt}{t^2(1+t)} = 6\int \frac{t^3 dt}{t+1} = 6\int \frac{t^3 + 1 - 1}{t+1}\,dt$$

$$= 6\int\left[(t^2 - t + 1) - \frac{1}{t+1}\right]dt$$

$$= 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + C$$

$$= 2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log(1 + x^{1/6}) + C$$

21. **(b)** Given, $\frac{dy}{dx} = \sin(x + y)\tan(x + y) - 1$

Put $x + y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$$\Rightarrow \quad \frac{dz}{dx} - 1 = \sin z \cdot \tan z - 1$$

$$\Rightarrow \quad \frac{dz}{dx} = \sin z \cdot \tan z$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{\sin^2 z}{\cos z}$$

$$\Rightarrow \quad \frac{\cos z}{\sin^2 z}\,dz = dx$$

On integrating both sides, we get $\int \frac{\cos z}{\sin^2 z}\,dz = \int dx$

Now, put $\sin z = t \Rightarrow \cos z\,dz = dt$

$$\Rightarrow \quad \int \frac{dt}{t^2} = \int dx \quad \Rightarrow \quad -\frac{1}{t} = x - C$$

$\Rightarrow -\operatorname{cosec} z = x - C \quad \Rightarrow x + \operatorname{cosec}(x + y) = C$

22. **(c)** The couple serve the committee in ${}^7C_3 \times {}^2C_2$ ways.

The couple does not serve the committee in ${}^7C_5$ ways.

$\therefore$ Required probability $= \frac{{}^7C_3 \times {}^2C_2 + {}^7C_5}{{}^9C_5} = \frac{56}{126} = \frac{4}{9}$

23. **(c)** Consider, $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$

$$= \cos^{-1}\left(\cos\left(2\pi - \frac{\pi}{3}\right)\right) + \sin^{-1}\left(\cos\left(2\pi - \frac{\pi}{3}\right)\right)$$

$$= \cos^{-1}\left(\cos\frac{\pi}{3}\right) + \sin^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$$

$$= \frac{\pi}{3} + \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right)\right)$$

$$= \frac{\pi}{3} + \sin^{-1}\left(\sin\frac{\pi}{6}\right)$$

$$= \frac{\pi}{3} + \frac{\pi}{6} = \frac{2\pi + \pi}{6} = \frac{\pi}{2}$$

24. **(a)** Here, we have,

$$n_1 = 35,\ \bar{x}_1 = 80,\ \sigma_1 = 4$$

and $\quad n_2 = 65,\ \bar{x}_2 = 70,\ \sigma_2 = 3$

Now, $\quad \bar{x}_{12} = \frac{35 \times 80 + 65 \times 70}{35 + 65} = 73.5$

and $\quad \sigma_{12} = \sqrt{\frac{35(16 + (6.5)^2) + 65(9 + (3.5)^2)}{100}}$

$$= \sqrt{\frac{35(16 + 42.25) + 65(9 + 12.25)}{100}} = 5.85$$

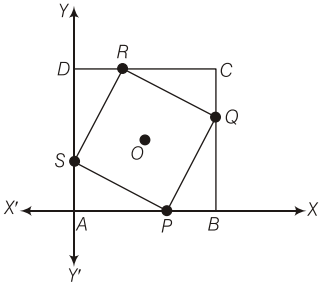
25. **(c)** Consider, the given series is

$$\log_9 3 + \log_{27} 3 - \log_{81} 3 + \log_{243} 3 \ldots$$
$$= \log_{3^2} 3 + \log_{3^3} 3 - \log_{3^4} 3 + \log_{3^5} 3 \ldots$$
$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots$$
$$= 1 - 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots$$
$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots$$
$$= \log_e(1 + 1) = \log_e 2$$

26. **(c)** In old cartesian coordinate system, coordinates of P and Q are $\left(\frac{2}{3}, 0\right)$ and $\left(1, \frac{2}{3}\right)$, respectively. Also, coordinates of centre of square $ABCD$ are $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Now, if origin is shifted to $\left(\frac{1}{2}, \frac{1}{2}\right)$ without rotation of axis, then coordinates of P and Q in new coordinate system are $\left(\frac{2}{3} - \frac{1}{2}, 0 - \frac{1}{2}\right)$ and $\left(1 - \frac{1}{2}, \frac{2}{3} - \frac{1}{2}\right)$

i.e. $\left(\frac{1}{6}, -\frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{1}{6}\right)$, respectively.



Now, in new coordinate system,

$$PQ = \sqrt{\left(\frac{1}{2} - \frac{1}{6}\right)^2 + \left(\frac{1}{6} + \frac{1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$$

Hence, area of square $PQRS$ in new coordinate system

$$= \left(\sqrt{\frac{5}{9}}\right)^2 = \frac{5}{9}$$

27. **(a)** $\sin^{-1}(\sin 5) > x^2 - 4x$

$\Rightarrow \quad \sin^{-1}[\sin(5 - 2\pi)] > x^2 - 4x$

$x^2 - 4x < 5 - 2\pi \Rightarrow x^2 - 4x + (2\pi - 5) < 0$

$2 - \sqrt{9 - 2\pi} < x < 2 + \sqrt{9 - 2\pi}$

28. **(c)** $\because \alpha, \beta$ and γ are the roots of $x^3 + 4x + 1 = 0$

$\therefore \alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \gamma\alpha = 4, \alpha\beta\gamma = -1$

Now, $\frac{\alpha^2}{\beta + \gamma} + \frac{\beta^2}{\gamma + \alpha} + \frac{\gamma^2}{\alpha + \beta} = -(\alpha + \beta + \gamma) = 0$

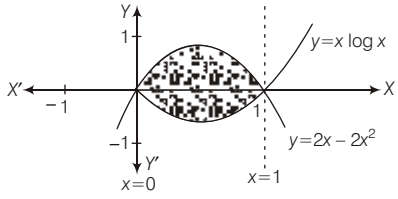
$$\frac{\alpha^2\beta^2}{(\beta + \gamma)(\gamma + \alpha)} + \frac{\beta^2\gamma^2}{(\gamma + \alpha)(\alpha + \beta)} + \frac{\gamma^2\alpha^2}{(\beta + \gamma)(\alpha + \beta)}$$

$$= \alpha\beta + \beta\gamma + \gamma\alpha = 4$$

and $\frac{\alpha^2\beta^2\gamma^2}{(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)} = -\alpha\beta\gamma = 1$

$\therefore$ Required equation is $x^3 + 4x - 1 = 0$.

29. **(c)** Required area $= \int_0^1 (2x - 2x^2 - x\log x)\,dx$



$$= \left[x^2 - \frac{2x^3}{3}\right]_0^1 - \left[\log x \cdot \frac{x^2}{2} - \frac{x^2}{4}\right]_0^1 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

30. **(c)** Given, $\lim_{n \to \infty} \frac{1 - (10)^n}{1 + (10)^{n+1}} = \frac{-\alpha}{10}$

$$\Rightarrow \quad \lim_{n \to \infty} \frac{10^n\left(\frac{1}{10^n} - 1\right)}{10^{n+1}\left(\frac{1}{10^{n+1}} + 1\right)} = \frac{-\alpha}{10}$$

$$\Rightarrow \quad \frac{1}{10}\lim_{n \to \infty} \frac{\left(\frac{1}{10^n} - 1\right)}{\left(\frac{1}{10^{n+1}} + 1\right)} = \frac{-\alpha}{10}$$

$$\Rightarrow \quad -\frac{1}{10} = \frac{-\alpha}{10} \quad \Rightarrow \quad \alpha = 1$$

31. **(d)** Clearly, $f'(x) = 6x^2 - 6x + 90$

$\because \quad D = b^2 - 4ac = 36 - 4 \times 6 \times 90 < 0$

$\therefore \quad f'(x) > 0 \ \forall \ x \in (-\infty, \infty)$

$\Rightarrow f(x)$ is increasing in $(-\infty, \infty)$.

32. **(b)** Let $\mathbf{OA} = \hat{i} - \hat{j}$ and $\mathbf{OB} = 2\hat{i} + 3\hat{j}$. Then, $\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$

$$= 2\hat{i} + 3\hat{j} - \hat{i} + \hat{j} = \hat{i} + 4\hat{j}$$

Now, vector perpendicular to $\mathbf{AB} = \lambda(4\hat{i} - \hat{j})$

and unit vector perpendicular to $\mathbf{AB} = \frac{4\hat{i} - \hat{j}}{\sqrt{17}}$

But unit normal vector is pointing towards origin

$\therefore$ Required unit normal vector $= \frac{-4\hat{i} + \hat{j}}{\sqrt{17}}$

33. **(d)** We have, $\frac{1}{e^{3x}}(e^x + e^{5x}) = a_0 + a_1x + a_2x^2 + \ldots$

$\Rightarrow \quad (e^{-2x} + e^{2x}) = a_0 + a_1x + a_2x^2 + \ldots$

$\Rightarrow \quad 2\left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \ldots\right] = a_0 + a_1x + a_2x^2 + \ldots$

$\Rightarrow \quad a_1 = a_3 = a_5 = \ldots = 0$

Hence, $2a_1 + 2^3a_3 + 2^5a_5 + \ldots = 0$

34. **(b)** Let $I = \int_1^e 10^{\log_e x}\,dx$

Now, put $\log_e x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt$

Also, when $x = 1, t = 0$ and when $x = e, t = 1$

$\therefore \quad I = \int_0^1 10^t \cdot e^t\,dt$

$$= \left[e^t \cdot \frac{10^t}{\log_e 10} - \int e^t \cdot \frac{10^t}{\log_e 10}\,dt\right]_0^1$$

$$= \left[e^t \cdot \frac{10^t}{\log_e 10}\right]_0^1 - \frac{1}{\log_e 10}\int_0^1 e^t \cdot 10^t\,dt$$

$$\Rightarrow \quad I + \frac{1}{\log_e 10} I = \frac{e \cdot 10}{\log_e 10} - \frac{1}{\log_e 10}$$

$$\Rightarrow \quad \left(\frac{1 + \log_e 10}{\log_e 10}\right) I = \frac{10 \cdot e - 1}{\log_e 10}$$

$$\Rightarrow \quad I = \frac{10 \cdot e - 1}{1 + \log_e 10}$$

$$= \frac{10 \cdot e - 1}{\log_e(10 \cdot e)}$$

35. **(a)** Let E_1 be the event that the bag contains 3 black balls and 3 balls of different colours.

E_2 be the event that the bag contains 4 black balls and 2 balls of different colours.

E_3 be the event that the bag contains 5 black balls and 1 ball of different colour.

E_4 be the event that the bag contains 6 black balls

and B be the event of getting 3 black balls in a draw of 3 balls.

Clearly, E_1, E_2, E_3 and E_4 are equally probable.

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

Also, $P\left(\frac{B}{E_1}\right) = \frac{{}^3C_3}{{}^6C_3} = \frac{1}{20}$

$$P\left(\frac{B}{E_2}\right) = \frac{{}^4C_3}{{}^6C_3} = \frac{4}{20} = \frac{1}{5}$$

$$P\left(\frac{B}{E_3}\right) = \frac{{}^5C_3}{{}^6C_3} = \frac{10}{20} = \frac{1}{2}$$

and $P\left(\frac{B}{E_4}\right) = \frac{{}^6C_3}{{}^6C_3} = \frac{20}{20} = 1$

Now, required probability

$$P\left(\frac{E_1}{B}\right) = \frac{P(E_1)\cdot P\left(\frac{B}{E_1}\right)}{P(E_1)\cdot P\left(\frac{B}{E_1}\right) + P(E_2)\cdot P\left(\frac{B}{E_2}\right) + P(E_3)\cdot P\left(\frac{B}{E_3}\right) + P(E_4)\cdot P\left(\frac{B}{E_4}\right)}$$

$$= \frac{\frac{1}{4}\cdot\frac{1}{20}}{\frac{1}{4}\cdot\frac{1}{20} + \frac{1}{4}\cdot\frac{1}{5} + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot 1}$$

$$= \frac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{5} + \frac{1}{2} + 1} = \frac{\frac{1}{20}}{\frac{35}{20}} = \frac{1}{35}$$

36. **(a)** Equation of plane passing through $(2, -1, -3)$ is

$$a(x-2) + b(y+1) + c(z+3) = 0 \quad \ldots(i)$$

Now, as given lines are parallel to it

$\therefore$ $3a + 2b - 4c = 0$ $\ldots(ii)$

and $2a - 3b + 2c = 0$ $\ldots(iii)$

Eliminating a, b and c from Eqs. (i), (ii) and (iii), we get

$$\begin{vmatrix} x-2 & y+1 & z+3 \\ 3 & 2 & -4 \\ 2 & -3 & 2 \end{vmatrix} = 0$$

$\Rightarrow$ $8x + 14y + 13z + 37 = 0$

37. **(c)** The point of intersection of lines $x - 2y = 1$ and $x + 3y = 2$ is $\left(\frac{7}{5}, \frac{1}{5}\right)$.

Since, required line is parallel to $3x + 4y = 0$

$\therefore$ Slope of required line $= -\frac{3}{4}$

Now, equation of line, whose slope is $-\frac{3}{4}$ and passes through $\left(\frac{7}{5}, \frac{1}{5}\right)$, is

$$20y - 4 = -15x + 21 \Rightarrow 3x + 4y - 5 = 0$$

38. **(c)** Clearly, mean $(\bar{x}) = \frac{1 + 2 + 3 + \ldots + (2n+1)}{(2n+1)}$

$$= \frac{(2n+1)(2n+1+1)}{2(2n+1)} = (n+1)$$

and $\sigma^2 = \frac{1}{(2n+1)}\sum_{r=0}^{2n}\{(1+r) - (n+1)\}^2$

$$= \frac{2}{(2n+1)}\{1^2 + 2^2 + 3^2 + \ldots + n^2\}$$

$$= \frac{2}{(2n+1)}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)}{3}$$

$\therefore$ $\sigma = \sqrt{\frac{n(n+1)}{3}}$

39. **(a)** We know that, each subset of $A \times B$ will form a relation. Therefore, number of relations from A to B is same as number of subset of $A \times B$.

$\because$ $n(A \times B) = n(A) \times n(B) = mn$

$\therefore$ Number of subset of $A \times B = 2^{mn}$

Hence, number of relation from A to $B = 2^{mn}$.

40. **(a)** Let us form the following truth table.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

It is clear from the last column that $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

41. **(d)** According to given information, we have the following table

Types of items	Working time on machine	Man labour
Shirt (x)	2 h	3 h
Pant (y)	3 h	2 h
Availability	70 h	75 h

Clearly, linear constraints are $2x + 3y \le 70$, $3x + 2y \le 75$ and $x, y \ge 0$.

42. **(b)** We have, $y = e^x(A\cos x + B\sin x)$

$$\Rightarrow \frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

$$= y + e^x(-A\sin x + B\cos x) \quad \ldots(i)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y - e^x(A\cos x + B\sin x) \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\frac{dy}{dx} - y - y = 2\frac{dy}{dx} - 2y$$

43. **(a)** Since, $\sin A$, $\sin B$, $\cos A$ are in GP.

$\therefore$ $\sin^2 B = \sin A \cdot \cos A$ $\ldots(i)$

For nature of roots of $x^2 + 2x\cot B + 1 = 0$,

Consider, $b^2 - 4ac = 4\cot^2 B - 4 = \frac{4\cos^2 B - 4\sin^2 B}{\sin^2 B}$

$$= \frac{4 - 8\sin^2 B}{\sin^2 B}$$

$$= \frac{4(1 - 2\sin A\cos A)}{\sin^2 B} \quad \text{[using Eq. (i)]}$$

$$= \frac{4(\sin A - \cos A)^2}{\sin^2 B} = 4\left(\frac{\sin A - \cos A}{\sin B}\right)^2$$

$\therefore$ Roots of given equation are always real.

44. **(c)** Clearly, $R = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$

$\Rightarrow$ $R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

Now, $RoR^{-1} = \{(3, 3), (3, 5), (5, 3), (5, 5)\}$

45. **(a)** The total number of points is 15. From these 15 points we can obtain ${}^{15}C_3$ triangles. However, if all the 3 points are chosen on the same straight line, we do not get a triangle.

$\therefore$ Required number of triangles $= {}^{15}C_3 - 3({}^5C_3) = 425$

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Mock Questions with Complete Solutions

Instructions

- This test consists of 45 questions and each question is allotted 3 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. $\int \frac{x}{(x^2-a^2)(x^2-b^2)}dx$ is equal to

(a) $\frac{1}{a^2-b^2}\log\left|\frac{x^2-a^2}{x^2-b^2}\right|+C$ (b) $\frac{1}{a^2-b^2}\log\left|\frac{x^2-b^2}{x^2-a^2}\right|+C$

(c) $\frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-a^2}{x^2-b^2}\right|+C$ (d) $\frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-b^2}{x^2-a^2}\right|+C$

2. $\lim_{n\to\infty}\frac{(n!)^{1/n}}{n}$ equals

(a) e (b) e^{-1}

(c) 1 (d) None of these

3. If $\frac{z-1}{z+1}$ is purely imaginary number $(z\neq -1)$, then $|z|$ is equal to

(a) 1 (b) 2

(c) 3 (d) 5

4. If α and β are roots of $x^2+ax-b=0$ and γ, δ are roots of $x^2+ax+b=0$, then $(\alpha-\gamma)(\beta-\delta)(\alpha-\delta)(\beta-\gamma)$ is equal to

(a) $2b$ (b) $2b^2$

(c) $4b^2$ (d) b

5. A GP consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occuping odd places, the common ratio will be equal to

(a) 2 (b) 3

(c) 4 (d) 5

6. If x^n-1 is divisible by $x-k$. Then, the least positive integral value of k is

(a) 1 (b) 2

(c) 3 (d) 4

7. If the slope of one line is double the slope of another line and the combined equation of the pair of lines is $\frac{x^2}{a}+\frac{2xy}{h}+\frac{y^2}{b}=0$, then $ab:h^2$ is equal to

(a) 9 : 8 (b) 3 : 2

(c) 8 : 3 (d) None of these

8. The length of the latusrectum of the parabola whose focus is $\left(\frac{u^2}{2g}\sin 2\alpha, \frac{-u^2}{2g}\cos 2\alpha\right)$ and directrix is $y=\frac{u^2}{2g}$, is

(a) $\frac{u^2}{g}\cos^2\alpha$ (b) $\frac{u^2}{g}\cos 2\alpha$

(c) $\frac{2u^2}{g}\cos 2\alpha$ (d) $\frac{2u^2}{g}\cos^2\alpha$

9. If the tangent line to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ make intercepts h and k with axes, then $\frac{a^2}{h^2}+\frac{b^2}{k^2}$ is equal to

(a) 0 (b) 1 (c) –1 (d) 2

10. Given that $\pi<\alpha<\frac{3\pi}{2}$, then the expression $\sqrt{(4\sin^4\alpha+\sin^2 2\alpha)}+4\cos^2\left(\frac{1}{4}\pi-\frac{1}{2}\alpha\right)$ is equal to

(a) 2 (b) $2+4\sin\alpha$

(c) $2-4\sin\alpha$ (d) None of these

11. Which of the following statements is not true?

(a) $A-(A-B)=B\cap A$

(b) $A-(A\cap B')=A'\cap B$

(c) $A\cup B-((A\cap B')\cup(B\cap A'))=A\cap B$

(d) $A-(A-(A\cap B))=A\cap B$

12. Let R be a relation on the set of natural numbers N defined by nRm, iff n is a factor of m, (i.e. $n|m$). Then, R is
(a) reflexive and symmetric
(b) transitive and symmetric
(c) equivalence
(d) reflexive, transitive but not symmetric

13. The solution of $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ is
(a) $3x(1+y^2)=4y^3+C$
(b) $3y(1+x^2)=4x^3+C$
(c) $3x(1-y^2)=4y^3+C$
(d) $3y(1+y^2)=4x^3+C$

14. Which of the following function have period 2π ?
(a) $y=\sin\left(2\pi t+\frac{\pi}{3}\right)+2\sin\left(3\pi t+\frac{\pi}{4}\right)+3\sin 5\pi t$
(b) $y=\sin\frac{\pi}{3}t+\sin\frac{\pi}{4}t$
(c) $y=\sin t+\cos 2t$
(d) None of the above

15. The shortest distance between two lines, one joining the points $(-1, 2, -3)$ and $(-16, 6, 4)$ and the other joining the points $(1, -1, 3)$ and $(4, 9, 7)$ is
(a) 3 *(b)* 2
(c) 7 *(d)* None of these

16. The value of the sum of the series $3\cdot{}^nC_0-8\cdot{}^nC_1+13\,{}^nC_2-18\cdot{}^nC_3+\ldots$ upto $(n+1)$ terms is
(a) 0 *(b)* 3^n
(c) 5^n *(d)* None of these

17. The value of the integral $\int_0^2|x^2-1|dx$ is
(a) 0 *(b)* 2
(c) $-\frac{1}{3}$ *(d)* –2

18. If three natural numbers from 1 to 100 are selected randomly, then probability that all are divisible by both 2 and 3, is
(a) $\frac{4}{105}$ *(b)* $\frac{4}{33}$
(c) $\frac{4}{35}$ *(d)* $\frac{4}{1155}$

19. If $f(x)=\begin{cases}\frac{\sin 5x}{x^2+2x}, & x\neq 0\\ k+\frac{1}{2}, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is
(a) 1 *(b)* –2
(c) 2 *(d)* $\frac{1}{2}$

20. $\lim\limits_{x\to\infty}(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x})$ is equal to
(a) $\frac{1}{2}$ *(b)* 0
(c) 1 *(d)* None of these

21. Let $[x]$ represents the greatest integer less than or equal to x, then the value of the determinant $\begin{vmatrix}[e] & [\pi] & [\pi^2-6]\\ [\pi] & [\pi^2-6] & [e]\\ [\pi^2-6] & [e] & [\pi]\end{vmatrix}$ is
(a) –8 *(b)* 8
(c) 10 *(d)* None of these

22. The sum of the infinite series $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)-\frac{1}{4}\left(\frac{1}{3^2}+\frac{1}{4^2}\right)+\frac{1}{6}\left(\frac{1}{3^3}+\frac{1}{4^3}\right)-\ldots$ is equal to
(a) $\frac{1}{2}\log 2$ *(b)* $\log\frac{3}{5}$
(c) $\log\frac{5}{3}$ *(d)* $\frac{1}{2}\log\frac{5}{3}$

23. $\lim\limits_{x\to 1}\frac{\tan(x^2-1)}{x-1}$ is equal to
(a) 2 *(b)* $\frac{1}{2}$ *(c)* –2 *(d)* $-\frac{1}{2}$

24. Bag A contains 4 green and 3 red balls and bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B, is
(a) $\frac{2}{7}$ *(b)* $\frac{2}{3}$ *(c)* $\frac{3}{7}$ *(d)* $\frac{1}{3}$

25. The standard deviation for the scores 1, 2, 3, 4, 5, 6 and 7 is 2. Then, the standard deviation of 12, 23, 34, 45, 56, 67 and 78 is
(a) 2 *(b)* 4 *(c)* 22 *(d)* 11

26. If $\sin\theta=\sin 15^\circ+\sin 45^\circ$, where $0^\circ<\theta<90^\circ$, then θ is equal to
(a) 45° *(b)* 54°
(c) 75° *(d)* 72°

27. Which of the following statements is a tautology?
(a) $(\sim q\wedge p)\wedge q$ *(b)* $(\sim q\wedge p)\wedge(p\wedge\sim p)$
(c) $(\sim q\wedge p)\vee(p\vee\sim p)$ *(d)* $(p\wedge q)\wedge(\sim(p\wedge q))$

28. If the equation, $12x^2-10xy+2y^2+11x-5y+k=0$ represents two straight lines, then the value of k is
(a) 1 *(b)* 2 *(c)* 0 *(d)* 3

29. The sum of the series $\frac{3}{4}+\frac{15}{16}+\frac{63}{64}+\ldots$ to n terms is equal to
(a) $n-\frac{4^n}{3}-\frac{1}{3}$ *(b)* $n+\frac{4^{-n}}{3}-\frac{1}{3}$
(c) $n+\frac{4^n}{3}-\frac{1}{3}$ *(d)* $n-\frac{4^{-n}}{3}+\frac{1}{3}$

30. The values of a for which the system of equations $x+y+z=0$, $x+ay+az=0$, $x-ay+z=0$ possesses non-zero solutions, are
(a) 1, 2 *(b)* 1, –1
(c) 1, 0 *(d)* None of these

31. If $\mathbf{b}$ and $\mathbf{c}$ are any two non-collinear unit vectors and $\mathbf{a}$ is any vector, then $(\mathbf{a}\cdot\mathbf{b})\mathbf{b}+(\mathbf{a}\cdot\mathbf{c})\mathbf{c}+\dfrac{\mathbf{a}(\mathbf{b}\times\mathbf{c})}{|\mathbf{b}\times\mathbf{c}|}\cdot(\mathbf{b}\times\mathbf{c})$ is equal to

(a) 0 *(b)* $\mathbf{a}$ *(c)* $\mathbf{b}$ *(d)* $\mathbf{c}$

32. Given, $f(0)=0$ and $f(x)=\dfrac{1}{(1-e^{-1/x})}$ for $x\neq 0$. Then, only one of the following statements on $f(x)$ is true, i.e. $f(x)$ is

(a) continuous at $x=0$
(b) not continuous at $x=0$
(c) both continuous and differentiable at $x=0$
(d) continuous but not differentiable at $x=0$

33. On the curve $x^3=12y$, the abscissa changes at a faster rate than the ordinate. Then, x belongs to the interval

(a) $(-4, 4)$ *(b)* $(-3, 3)$
(c) $(-2, 2)$ *(d)* None of these

34. If $\log_9(x^2-5x+6)>\log_3(x-4)$, x belongs to

(a) $(-\infty, 4)$ *(b)* $(4, \infty)$
(c) $(-\infty, -4)\cup(4, \infty)$ *(d)* No real values of x

35. If two equations $x^2+a^2=1-2ax$ and $x^2+b^2=1-2bx$ have only one common root, then

(a) $a-b=1$ *(b)* $a-b=-1$
(c) $a-b=2$ *(d)* $|a-b|=1$

36. Let the function $g:(-\infty,\infty)\to\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ be given $g(u)=2\tan^{-1}(e^u)-\dfrac{\pi}{2}$. Then, g is

(a) even and strictly increasing in $(0, \infty)$
(b) odd and strictly decreasing in $(-\infty, \infty)$
(c) odd and strictly increasing in $(-\infty, \infty)$
(d) neither even nor odd, but strictly increasing in $(-\infty, \infty)$

37. The domain of the function $f(x)=\sqrt{\dfrac{-\log_{0.3}(x-1)}{-x^2+3x+18}}$ is

(a) $[2, 6]$ *(b)* $(2, 6)$
(c) $[2, 6)$ *(d)* None of these

38. The area bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$, is

(a) $\left(\dfrac{5\pi}{4}-2\right)$ sq units *(b)* $\dfrac{(5\pi-2)}{4}$ sq units
(c) $\dfrac{(5\pi-2)}{2}$ sq units *(d)* $\left(\dfrac{\pi}{2}-5\right)$ sq units

39. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated, is

(a) 2^{10} *(b)* 101
(c) 1023 *(d)* 10^2

40. The maximum value of $Z=4x+2y$ subject to the constraints $2x+3y\le 18$, $x+y\ge 10$; $x, y\ge 0$ is

(a) 20 *(b)* 33
(c) 40 *(d)* None of these

41. The coefficient of x^n in the expansion of $\dfrac{(a+bx+cx^2)}{e^x}$ is

(a) $\dfrac{a(-1)^n}{n!}+\dfrac{b(-1)^{n-1}}{(n-1)!}+\dfrac{c(-1)^{n-2}}{(n-2)!}$

(b) $\dfrac{a}{n!}+\dfrac{b}{(n-1)!}+\dfrac{c}{(n-2)!}$

(c) $\dfrac{(-1)^n}{n!}+\dfrac{(-1)^{n-1}}{(n-1)!}+\dfrac{(-1)^{n-2}}{(n-2)!}$

(d) $\dfrac{(-1)^n}{n!}-\dfrac{(-1)^{n-1}}{(n-1)!}+\dfrac{(-1)^{n-2}}{(n-2)!}$

42. In a town of 840 persons, 450 persons read Hindi newspaper, 300 read English newspaper and 200 read both. Then, the number of persons who read neither of the newspapers, is

(a) 210 *(b)* 290 *(c)* 180 *(d)* 260

43. The differential equation representing the family of curves $y=xe^{cx}$ (c is a constant) is

(a) $\dfrac{dy}{dx}=\dfrac{y}{x}\left(1-\log\dfrac{y}{x}\right)$ *(b)* $\dfrac{dy}{dx}=\dfrac{y}{x}\log\left(\dfrac{y}{x}\right)+1$

(c) $\dfrac{dy}{dx}=\dfrac{y}{x}\left(1+\log\dfrac{y}{x}\right)$ *(d)* $\dfrac{dy}{dx}+1=\dfrac{y}{x}\log\left(\dfrac{y}{x}\right)$

44. If $\tan^{-1}y=4\tan^{-1}x$, then $\dfrac{1}{y}$ is zero for

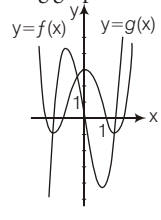
(a) $x=1\pm\sqrt{2}$ *(b)* $x=\sqrt{2}\pm\sqrt{3}$
(c) $x=3\pm2\sqrt{2}$ *(d)* all values of x

45. The lines $(lx+my)^2-3(mx-ly)^2=0$ and $lx+my+n=0$ form

(a) an isosceles triangle
(b) a right angled triangle
(c) an equilateral triangle
(d) None of the above

Think About it!

If the functions f and g have following graph



then g is derivative of f. "It is true".

Answers *with* Explanation

1. **(c)** Let $I = \int \frac{xdx}{(x^2 - a^2)(x^2 - b^2)}$

Put $x^2 = t$ and $2xdx = dt$

$\therefore \quad I = \frac{1}{2}\int \frac{dt}{(t - a^2)(t - b^2)}$

$\Rightarrow \quad I = \frac{1}{2}\int \left\{\frac{1}{(a^2 - b^2)(t - a^2)} - \frac{1}{(a^2 - b^2)(t - b^2)}\right\}dt$

$\Rightarrow \quad I = \frac{1}{2(a^2 - b^2)}\log\left|\frac{x^2 - a^2}{x^2 - b^2}\right| + C$

2. **(b)** Let $A = \lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n}$. Then, $\log A = \lim_{n\to\infty}\left\{\frac{1}{n}\log\left(\frac{1\cdot 2\cdot 3 \ldots n}{n^n}\right)\right\}$

$\Rightarrow \quad \log A = \lim_{n\to\infty}\left\{\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)\right\} = \int_0^1 \log x\, dx = -1$

$\Rightarrow \quad A = e^{-1}$

3. **(a)** Since, $\frac{z-1}{z+1}$ is purely imaginary

$\therefore \quad \left(\frac{z-1}{z+1}\right) = -\overline{\left(\frac{z-1}{z+1}\right)} \Rightarrow \frac{z-1}{z+1} = -\left(\frac{\bar{z}-1}{\bar{z}+1}\right)$

Using componendo and dividendo, we get

$z\bar{z} = |z|^2 = 1$

$\therefore \quad |z| = 1$

4. **(c)** Let $E = (\alpha - \gamma)(\beta - \delta)(\alpha - \delta)(\beta - \gamma)$

$= (\alpha^2 - (\gamma + \delta)\alpha + \gamma\delta)(\beta^2 - (\gamma + \delta)\beta + \gamma\delta)$

$= (\alpha^2 - (\alpha + \beta)\alpha - \alpha\beta)(\beta^2 - (\alpha + \beta)\beta - \alpha\beta)$

$[\because \alpha + \beta = \gamma + \delta = -a \text{ and } \gamma\delta = -\alpha\beta = b]$

$\Rightarrow \quad E = 4(\alpha\beta)^2 = 4b^2$

5. **(c)** Let there be $2n$ terms in the given GP with first term a and the common ratio r. Then, sum of all terms = 5 (sum of terms occuping odd places)

$\Rightarrow \quad a + ar + ar^2 + \ldots + ar^{2n-1}$

$= 5(a + ar^2 + \ldots + ar^{2n-2})$

$\Rightarrow \quad \frac{a(r^{2n} - 1)}{r - 1} = 5a\frac{(r^{2n} - 1)}{r^2 - 1}$

$\Rightarrow \quad r + 1 = 5$

$\Rightarrow \quad r = 4$

6. **(a)** Let $P(n)$ be the given statement, i.e.

$P(n) : x^n - 1$ is divisible by $x - k$.

Now, for $n = 1$, $P(1) : x - 1$ is divisible by $x - k$, which is true by given condition.

Thus, the least value of k for which $P(1)$ is true, is 1.

7. **(a)** If $m, 2m$ are slopes, then

$$m + 2m = -\frac{\frac{2}{h}}{\frac{1}{b}} = -\frac{2b}{h}$$

and $$m\cdot 2m = \frac{\frac{1}{a}}{\frac{1}{b}} = \frac{b}{a}$$

Eliminating m, we get

$$2\left(\frac{-2b}{3h}\right)^2 = \frac{b}{a}$$

$\therefore \quad \frac{ab}{h^2} = \frac{9}{8}$

8. **(d)** Since, length of latusrectum of the parabola is twice the distance between the focus and the directrix, therefore we have

$$\text{Length of latusrectum} = 2\left\{\frac{\left|-\frac{u^2}{2g}\cos 2\alpha - \frac{u^2}{2g}\right|}{\sqrt{1^2 + 0^2}}\right\}$$

$$= \frac{2u^2}{g}\cos^2\alpha$$

9. **(b)** Clearly, the equation of tangent at the point (x_1, y_1) be $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

$\Rightarrow \quad \frac{x}{\frac{a^2}{x_1}} + \frac{y}{\frac{b^2}{y_1}} = 1,$

which meets axes at $\left(\frac{a^2}{x_1}, 0\right)$ and $\left(0, \frac{b^2}{y_1}\right)$

but $\frac{a^2}{x_1} = h$

and $\frac{b^2}{y_1} = k$

$\Rightarrow \quad x_1 = \frac{a^2}{h}, y_1 = \frac{b^2}{k}$

$\Rightarrow \quad \frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$

10. **(a)** Given expression

$= \sqrt{[4\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)]} + 2\left[1 + \cos\left(\frac{1}{2}\pi - \alpha\right)\right]$

$= 2\sqrt{(\sin^2\alpha)} + 2(1 + \sin\alpha)$

$= 2(-\sin\alpha) + (2 + 2\sin\alpha) = 2$

$[\because \pi < \alpha < \frac{3\pi}{2} \Rightarrow \sin\alpha < 0 \Rightarrow -\sin\alpha > 0]$

11. **(b)** $A - (A - B) = A - (A \cap B^C) = (A - A) \cup (A - B^C) = A \cap B$

$\therefore$ Option (a) is true, but option (b) is not true as

$A - (A \cap B') = A \cap B \neq A' \cap B$

12. **(d)** Since, n is a factor of n, $\forall n \in N$, so R is reflexive.

Note that 2 is factor 4 but 4 is not, so $2R4$ but $4\not R2$.

Therefore, R is not symmetric.

Now, nRp, pRq, then

$p = nk_1, q = pk_2$ for some $k_1, k_2 \in N$

$\Rightarrow \quad q = k_1 \cdot k_2 \cdot n$

$\Rightarrow$ R is transitive.

13. **(b)** Given, $\frac{dy}{dx} + \frac{2x}{1 + x^2}\cdot y = \frac{4x^2}{1 + x^2}$

$\therefore$ IF $= e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1 + x^2)$

and solution is given by

$$y(1+x^2)=\int(1+x^2)\cdot\frac{4x^2}{1+x^2}dx$$

$$\Rightarrow \quad y(1+x^2)=\frac{4x^3}{3}+C_1$$

$$\Rightarrow \quad 3y(1+x^2)=4x^3+C$$

14. ***(c)*** The period of the function in option (a) is 2.
The period of the function in option (b) is 24.
The period of the function in option (c) is 2π.

15. ***(c)*** Let $A=(-1,2,-3)$, $B=(-16,6,4)$, $C=(1,-1,3)$ and $D=(4,9,7)$. Then, $\mathbf{AB}=-15\hat{\mathbf{i}}+4\hat{\mathbf{j}}+7\hat{\mathbf{k}}$

and $\mathbf{CD}=3\hat{\mathbf{i}}+10\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

Now, let $\mathbf{u}=\mathbf{AB}\times\mathbf{CD}$

$$=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -15 & 4 & 7\\ 3 & 10 & 4\end{vmatrix}$$

$$=\hat{\mathbf{i}}(-54)-\hat{\mathbf{j}}(-81)+\hat{\mathbf{k}}(-162)$$

$$=27(-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-6\hat{\mathbf{k}})$$

$$\Rightarrow \quad \hat{\mathbf{u}}=\frac{1}{7}(-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-6\hat{\mathbf{k}})$$

Now, shortest distance $=|\mathbf{AC}\cdot\hat{\mathbf{u}}|$

$$=\frac{1}{7}|(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+6\hat{\mathbf{k}})\cdot(-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-6\hat{\mathbf{k}})|$$

$$\Rightarrow \quad =\frac{49}{7}=7$$

16. ***(a)*** The general term of the given series is

$$T_{r+1}=(-1)^r(3+5r)\cdot{}^nC_r$$

$$\therefore \text{Required sum}=\sum_{r=0}^{n}(-1)^r(3+5r)\,{}^nC_r$$

$$=3\sum_{r=0}^{n}(-1)^r\,{}^nC_r+5\sum_{r=0}^{n}(-1)^r\,r\,{}^nC_r$$

$$=3(C_0-C_1+C_2-C_3+C_4-\ldots+(-1)^n\cdot C_n)$$

$$+5(-C_1+2C_2-3C_3+4C_4-\ldots+(-1)^n\cdot n\cdot C_n)$$

$$S=0+0=0$$

17. ***(b)*** $\int_0^2|x^2-1|dx=\int_0^1-(x^2-1)dx+\int_1^2(x^2-1)dx$

$$=\left[-\frac{x^3}{3}+x\right]_0^1+\left[\frac{x^3}{3}-x\right]_1^2$$

$$=-\frac{1}{3}+1+\frac{8}{3}-2-\frac{1}{3}+1=2$$

18. ***(d)*** Let $E=$ event of numbers divisible by 2 and 3

$$=(6,12,\ldots,96)$$

$$n(E)=16$$

$$\therefore \text{Required probability}=\frac{{}^{16}C_3}{{}^{100}C_3}=\frac{4}{1155}$$

19. ***(c)*** Clearly, LHL $=\lim_{h\to 0}f(0-h)$

$$=\lim_{h\to 0}\frac{\sin 5(0-h)}{(0-h)^2+2(0-h)}$$

$$=-\lim_{h\to 0}\frac{\sin 5h}{h^2-2h}$$

$$=-\lim_{h\to 0}\left(\frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)}\right)=\frac{5}{2}$$

Since, it is continuous at $x=0$.

$$\therefore \quad \text{LHL}=f(0)$$

$$\Rightarrow \quad \frac{5}{2}=k+\frac{1}{2}$$

$$\Rightarrow \quad k=2$$

20. ***(a)*** Consider, $\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

$$=\lim_{x\to\infty}\left(\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\times\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}$$

$$=\lim_{y\to 0}\frac{\frac{(\sqrt{1+\sqrt{y}})}{\sqrt{y}}}{\sqrt{\frac{1}{y}+\frac{\sqrt{1+\sqrt{y}}}{\sqrt{y}}}+\frac{1}{\sqrt{y}}} \qquad \left[\text{put } x=\frac{1}{y}\right]$$

$$=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{y}}}{\sqrt{1+\sqrt{y(1+\sqrt{y})}}+1}=\frac{1}{2}$$

21. ***(a)*** Let $\Delta=\begin{vmatrix}[e] & [\pi] & [\pi^2-6]\\ [\pi] & [\pi^2-6] & [e]\\ [\pi^2-6] & [e] & [\pi]\end{vmatrix}$

$$=\begin{vmatrix}2 & 3 & 3\\ 3 & 3 & 2\\ 3 & 2 & 3\end{vmatrix}$$

$$=2(9-4)-3(9-6)+3(6-9)$$

$$=-8$$

22. ***(d)*** Consider, $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)-\frac{1}{4}\left(\frac{1}{3^2}+\frac{1}{4^2}\right)+\frac{1}{6}\left(\frac{1}{3^3}+\frac{1}{4^3}\right)-\ldots$

$$=\frac{1}{2}\left[\frac{1}{3}-\frac{1}{2}\left(\frac{1}{3^2}\right)+\frac{1}{3}\left(\frac{1}{3^3}\right)-\ldots\right]$$

$$+\frac{1}{2}\left[\frac{1}{4}-\frac{1}{2}\left(\frac{1}{4^2}\right)+\frac{1}{3}\left(\frac{1}{4^3}\right)-\ldots\right]$$

$$=\frac{1}{2}\left[\log\left(1+\frac{1}{3}\right)\right]+\frac{1}{2}\left[\log\left(1+\frac{1}{4}\right)\right]=\frac{1}{2}\log\left(\frac{5}{3}\right)$$

23. ***(a)*** $\lim_{x\to 1}\frac{\tan(x^2-1)}{x-1}=\lim_{x\to 1}\frac{\sec^2(x^2-1)\cdot 2x}{1}$

$$=2\cdot\sec^2(0)=2$$

24. ***(c)*** $P\left(\frac{G}{A}\right)=\frac{{}^4C_1}{{}^7C_1}=\frac{4}{7}$ and $P\left(\frac{G}{B}\right)=\frac{{}^3C_1}{{}^7C_1}=\frac{3}{7}$

Now, $P\left(\frac{B}{G}\right)=\frac{P(B)\cdot P\left(\frac{G}{B}\right)}{P(A)P\left(\frac{G}{A}\right)+P(B)\,P\left(\frac{G}{B}\right)}$

$$=\frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{7}}=\frac{3}{7}$$

25. **(c)** Let $x_i = i$, where $i = 1, 2, 3, 4, 5, 6$ and 7
and let $y_1 = 12,\ y_2 = 23,\ y_3 = 34,\ y_4 = 45,$
$y_5 = 56,\ y_6 = 67,\ y_7 = 78$
Clearly, $y_i = 11 \cdot x_i + 1,\ \forall\ i = 1, 2, \ldots, 7$
$\therefore \quad \sigma_y = 11 \cdot \sigma_x$
Hence, required standard deviation $= 11 \cdot 2 = 22$

26. **(c)** We have, $\sin\theta = \sin 15° + \sin 45°$
$= 2\sin 30° \cos 15°$
$= 2 \cdot \frac{1}{2}\cos(90° - 75°) = \sin 75°$
$\Rightarrow \quad \theta = 75°$

27. **(c)** Let us form the following truth table

p	q	$\sim p$	$\sim q$	$\sim q \wedge p$	$p \wedge q$	$\sim(p \wedge q)$	$(\sim q \wedge p) \wedge q$	$(p \vee \sim p)$	$(p \wedge q) \wedge (\sim(p \wedge q))$	$(\sim q \wedge p) \vee (p \vee \sim p)$
T	T	F	F	F	T	F	F	T	F	T
T	F	F	T	T	F	T	F	T	F	T
F	T	T	F	F	F	T	F	T	F	T
F	F	T	T	F	F	T	F	T	F	T

It is clear from the table that the last column have all true values.

28. **(b)** Here, we have, $a = 12,\ b = 2,\ h = -5,\ f = -\frac{5}{2},\ g = \frac{11}{2},$
$c = k$
The given equation represents a pair of straight line, if
$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$
$$12 \cdot 2 \cdot k + 2\left(\frac{-5}{2}\right)\left(\frac{11}{2}\right)(-5) - 12\left(-\frac{5}{2}\right)^2 - 2\left(\frac{11}{2}\right)^2 - k(-5)^2 = 0$$
$$\Rightarrow \quad 24k + \frac{275}{2} - \frac{150}{2} - \frac{121}{2} - 25k = 0$$
$$\Rightarrow \quad k = 2$$

29. **(b)** For $n = 1$,
$$n - \frac{4^n}{3} - \frac{1}{3} = 1 - \frac{4}{3} - \frac{1}{3} = -\frac{2}{3}$$
$$n + \frac{4^{-n}}{3} - \frac{1}{3} = 1 + \frac{4^{-1}}{3} - \frac{1}{3} = \frac{3}{4}$$
$$n + \frac{4^n}{3} - \frac{1}{3} = 1 + \frac{4}{3} - \frac{1}{3} = 2$$
$$n - \frac{4^{-n}}{3} + \frac{1}{3} = 1 - \frac{4^{-1}}{3} + \frac{1}{3} = \frac{5}{4}$$
For $n = 2$,
$$n + \frac{4^{-n}}{3} - \frac{1}{3} = 2 + \frac{1}{48} - \frac{1}{3} = \frac{27}{16}$$
and $\quad \frac{3}{4} + \frac{15}{16} = \frac{27}{16}$

30. **(b)** The given system of equations possesses non-zero solution, if
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a \\ 1 & -a & 1 \end{vmatrix} = 0$$
$$\Rightarrow \quad \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & a-1 \\ 0 & -(a+1) & 0 \end{vmatrix} = 0$$
$$\Rightarrow \quad 1[0 + (a^2 - 1)] = 0$$
$$\Rightarrow \quad a^2 = 1$$
$$\Rightarrow \quad a = \pm 1$$

31. **(b)** Let $\mathbf{b} = \hat{\mathbf{i}}$ and $\mathbf{c} = \hat{\mathbf{j}}$. Then,
$$|\mathbf{b} \times \mathbf{c}| = |\hat{\mathbf{k}}| = 1$$
Now, let $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$. Then,
$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \hat{\mathbf{i}} = \mathbf{a}_1, \mathbf{a} \cdot \mathbf{c} = \mathbf{a} \cdot \hat{\mathbf{j}} = \mathbf{a}_2$$
and $\quad \mathbf{a} \cdot \frac{\mathbf{b} \times \mathbf{c}}{|\mathbf{b} \times \mathbf{c}|} = \mathbf{a} \cdot \hat{\mathbf{k}} = \mathbf{a}_3$
$$\therefore \quad (\mathbf{a} \cdot \mathbf{b})\mathbf{b} + (\mathbf{a} \cdot \mathbf{c})\mathbf{c} + \frac{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}{|\mathbf{b} \times \mathbf{c}|} \cdot (\mathbf{b} \times \mathbf{c})$$
$$= a_1\mathbf{b} + a_2\mathbf{c} + a_3(\mathbf{b} \times \mathbf{c})$$
$$= a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}} = \mathbf{a}$$

32. **(b)** At $x = 0$,
$$\text{LHL} = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{1}{1 - e^{\frac{-1}{(0-h)}}}$$
$$= \lim_{h \to 0} \frac{1}{1 - e^{1/h}}$$
$$= \lim_{h \to 0} \frac{e^{-1/h}}{e^{-1/h} - 1} = 0$$
and $\text{RHL} = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{1}{1 - e^{\frac{-1}{(0+h)}}}$
$$= \lim_{h \to 0} \frac{1}{1 - e^{-1/h}} = 1$$
$\therefore$ Function is not continuous at $x = 0$.

33. **(c)** By the given condition, we have $\left|\frac{dx}{dy}\right| > 1$.
Now, consider $x^3 = 12y$
$$\Rightarrow \quad 3x^2\frac{dx}{dy} = 12$$
$$\Rightarrow \quad \frac{dx}{dy} = \frac{4}{x^2}$$
$$\Rightarrow \quad \frac{4}{x^2} > 1$$
$$\Rightarrow \quad x^2 - 4 < 0$$
$$\Rightarrow \quad -2 < x < 2$$

34. **(b)** We have,
$$\log_9(x^2 - 5x + 6) > \log_3(x - 4)$$
$$\Rightarrow \quad \frac{\log_e(x^2 - 5x + 6)}{\log_e 9} > \frac{\log_e(x-4)}{\log_e 3}$$
$$\Rightarrow \quad \log_e(x^2 - 5x + 6) > \log_3 9 \cdot \log_e(x - 4)$$
$$\Rightarrow \quad \log_e(x^2 - 5x + 6) > \log_e(x-4)^2$$
$$\Rightarrow \quad x^2 - 5x + 6 > x^2 - 8x + 16$$
$$\Rightarrow \quad 3x - 10 > 0$$
$$\Rightarrow \quad x > \frac{10}{3} \qquad \ldots(i)$$
Also, $\quad x^2 - 5x + 6 > 0$
$$\Rightarrow \quad x > 3 \text{ or } x < 2 \qquad \ldots(ii)$$
and $\quad x - 4 > 0$
$$\Rightarrow \quad x > 4 \qquad \ldots(iii)$$
From Eqs. (i), (ii) and (iii), we get $x > 4$

35. **(c)** On comparing the given equation with $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ respectively, we get

$$a_1=1, b_1=2a, c_1=a^2-1$$

and $$a_2=1, b_2=2b, c_2=b^2-1$$

Condition for common roots is

$$(a_1c_2-a_2c_1)^2=(b_1c_2-b_2c_1)(a_1b_2-a_2b_1)$$

$$[1\cdot(b^2-1)-1\cdot(a^2-1)]^2=[2a\cdot(b^2-1)-2b(a^2-1)](2b-2a)$$

$$\Rightarrow \quad (b^2-a^2)^2=4(b-a)(b-a)(ab+1)$$

$$\Rightarrow \quad (b-a)^2=4$$

$$\Rightarrow \quad a-b=2$$

36. **(c)** We have, $g(u)=2\tan^{-1}(e^u)-\frac{\pi}{2}$, for $u\in(-\infty,\infty)$

Now, $$g(-u)=2\tan^{-1}(e^{-u})-\frac{\pi}{2}$$

$$=2\cot^{-1}(e^u)-\frac{\pi}{2}=2\left(\frac{\pi}{2}-\tan^{-1}(e^u)\right)-\frac{\pi}{2}$$

$$=\frac{\pi}{2}-2\tan^{-1}(e^u)=-g(u)$$

$\therefore g(u)$ is an odd function.

Also, $$g'(u)=\frac{2}{1+(e^u)^2}\cdot e^u-0>0$$

$\Rightarrow g(u)$ is strictly increasing.

37. **(c)** We have, $f(x)=\sqrt{\frac{-\log_{0.3}(x-1)}{-x^2+3x+18}}$

Clearly, $f(x)$ is defined, if $\frac{\log_{0.3}(x-1)}{x^2-3x-18}\geq 0$

i.e. if $\log_{0.3}(x-1)\geq 0$ and $x^2-3x-18>0$ …(i)

or

If $\log_{0.3}(x-1)\leq 0$ and $x^2-3x-18<0$ …(ii)

In case I $x^2-3x-18>0$

$\Rightarrow x<-3$ or $x>6$ and $\log_{0.3}(x-1)\geq 0\Rightarrow 0<x-1\leq 1$

$\Rightarrow 1<x\leq 2$

$\therefore$ No solution exist.

In case II $x^2-3x-18<0\Rightarrow -3<x<6$

and $\log_{0.3}(x-1)\leq 0$

$$\Rightarrow \quad x-1\geq 1$$

$$\Rightarrow \quad x\geq 2$$

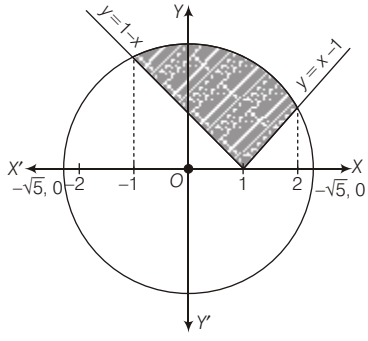
$$\Rightarrow \quad x\in[2,6)$$

Hence, domain of $f(x)=[2,6)$

38. **(b)** Given equation of curves are

$$y=\sqrt{5-x^2} \quad \text{and} \quad y=|x-1|$$

$$\Rightarrow \quad y^2+x^2=5 \quad \text{and} \quad y=|x-1|$$



Now, required area = area of shaded region

$$=\int_{-1}^{2}\sqrt{5-x^2}dx-\int_{-1}^{1}(1-x)dx-\int_{1}^{2}(x-1)dx$$

$$=\left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}-\left[x-\frac{x^2}{2}\right]_{-1}^{1}-\left[\frac{x^2}{2}-x\right]_{1}^{2}$$

$$=\frac{5}{2}\sin^{-1}(1)-\frac{1}{2}$$

$$=\frac{5\pi}{4}-\frac{1}{2}=\left(\frac{5\pi-2}{4}\right)\text{ sq units}$$

39. **(c)** The hall can be illuminated by switching on atleast one of the 10 bulbs. Therefore, the required number of ways is $2^{10}-1=1023$.

40. **(d)** Given, LPP is

Maximum $Z=4x+2y$

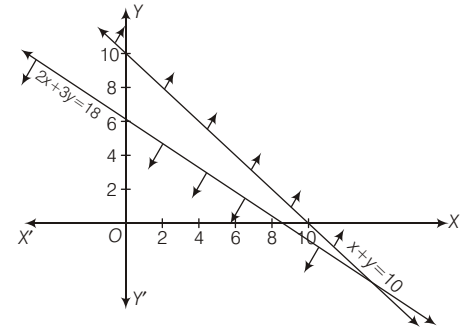
Subject to the constraints

$$2x+3y\leq 18$$

$$x+y\geq 10$$

$$x, y\geq 0$$

Let us draw the graph of above equations.



From the graph, it is clear that there is no common area. So, we cannot find maximum value of Z.

41. **(a)** Consider, $\frac{(a+bx+cx^2)}{e^x}=(a+bx+cx^2)e^{-x}$

$$=(a+bx+cx^2)\left(1-x+\frac{x^2}{2!}+\ldots+\frac{(-1)^n x^n}{n!}+\ldots\right)$$

$\therefore$ Coefficient of

$$x^n=\frac{a(-1)^n}{n!}+b\frac{(-1)^{n-1}}{(n-1)!}+\frac{c(-1)^{n-2}}{(n-2)!}$$

42. **(b)** Here, we have $n(U)=840$, $n(H)=450$, $n(E)=300$ and $n(E\cap H)=200$

$\therefore$ $$n(H\cup E)=n(H)+n(E)-n(E\cap H)$$

$$=450+300-200=550$$

Now, the number of persons who read neither of the newspaper

$$=n(\overline{H}\cap\overline{E})=n(U)-n(H\cup E)$$

$$=840-550=290$$

43. **(c)** We have, $y=xe^{cx}$

$$\Rightarrow \quad \frac{dy}{dx}=e^{cx}+x\cdot e^{cx}\cdot c=\frac{y}{x}+y\cdot c \quad \text{…(i)}$$

Now, consider $y=xe^{cx}$

On taking log both sides, we get

$$\log y=\log x+cx$$

$$\Rightarrow \quad c = \frac{1}{x}\log\left(\frac{y}{x}\right) \qquad \ldots(ii)$$

Thus, from Eqs. (i) and (ii), we get

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y}{x}\cdot\log\left(\frac{y}{x}\right)$$

$$= \frac{y}{x}\left[1+\log\left(\frac{y}{x}\right)\right]$$

44. ***(a)*** If we put $x = \tan\theta$, then the given equality becomes $\tan^{-1} y = 4\theta$

$$\Rightarrow \quad y = \tan 4\theta = \frac{2\tan 2\theta}{1-\tan^2 2\theta} = \frac{2\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]}{1-\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]^2}$$

$$= \frac{2\cdot 2x\cdot(1-x^2)}{(1-x^2)^2 - 4x^2}$$

$$= \frac{4x(1-x^2)}{x^4 - 6x^2 + 1}$$

Now, $\frac{1}{y}$ is zero, if $x^4 - 6x^2 + 1 = 0$

$$\Rightarrow \quad x^2 = (1\pm\sqrt{2})^2 \quad \Rightarrow \quad x = 1\pm\sqrt{2}$$

45. ***(c)*** Given lines are $[(l+\sqrt{3}m)x + (m-\sqrt{3}l)y]$ $[(l-\sqrt{3}m)x + (m+\sqrt{3}l)y] = 0$

and $lx + my + n = 0$

$$\therefore \quad m_1 = -\frac{(l+\sqrt{3}m)}{m-\sqrt{3}l},\ m_2 = -\frac{(l-\sqrt{3}m)}{(m+\sqrt{3}l)}$$

and $m_3 = -\frac{l}{m}$

$$\therefore \quad \theta_1 = \tan^{-1}\left|\frac{-\left(\frac{l+\sqrt{3}m}{m-\sqrt{3}l}\right)+\frac{l}{m}}{1+\left(\frac{l+\sqrt{3}m}{m-\sqrt{3}l}\right)\cdot\frac{l}{m}}\right| = 60°$$

$$\theta_2 = \tan^{-1}\left|\frac{-\left(\frac{l-\sqrt{3}m}{m+\sqrt{3}l}\right)+\frac{l}{m}}{1+\left(\frac{l-\sqrt{3}m}{m+\sqrt{3}l}\right)\left(\frac{l}{m}\right)}\right| = 60°$$

Hence, triangle is equilateral.

वस्तुनिष्ठ

मूलभूत शृंखला

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हिन्दी माध्यम की एकमात्र पुस्तक जिसकी अध्ययन सामग्री व प्रश्नों का कठिनता स्तर JEE MAIN & ADVANCED के अनुरूप है

MATHEMATICS SPECTRUM for You

In this section, we invite our readers to send their problems and we will try to provide the best possible solutions given by our expert panel.

Q1 Sir, I am unable to crack the following sum. 'Find the value of θ, if $\lim_{x \to 0} [1 + x \log (1 + b^2)]^{1/x} = 2b \sin^2 \theta$, where, $b > 0$ and $\theta \in (-\pi, \pi]$'. Please provide me its solution with detailed explanation. **[Rakhi, Lucknow]**

➥ Dear Rakhi, when we substitute $x = 0$ in the LHS of given problem, we get an indeterminate form of the type 1^{∞}. To overcome with this indeterminate form, you should aware about the result stated below

'If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$, then

$$\lim_{x \to a} \{1 + f(x)\}^{1/g(x)} = e^{\lim_{x \to a} \frac{f(x)}{g(x)}},$$

Now, consider

$$\text{LHS} = \lim_{x \to 0} [1 + x \log (1 + b^2)]^{1/x}$$

$$= e^{\lim_{x \to 0} \{x \log (1 + b^2)\} \cdot \frac{1}{x}}$$

$$= e^{\log (1 + b^2)}$$

$$= 1 + b^2$$

Thus, from given equation, we have

$$1 + b^2 = 2b \sin^2 \theta$$

$$\Rightarrow \quad \sin^2 \theta = \frac{b^2 + 1}{2b} \qquad \ldots(i)$$

Now, as we know that AM $\geq$ GM

$$\therefore \quad \frac{b + \frac{1}{b}}{2} \geq \left(b \cdot \frac{1}{b}\right)^{1/2}$$

$$\Rightarrow \quad \frac{b^2 + 1}{2b} \geq 1 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\sin^2 \theta = 1$$

$$\Rightarrow \quad \theta = \pm \frac{\pi}{2}$$

as $\quad \theta \in (-\pi, \pi]$

Q2 Sir, please provide me best solution of the following question.

'In how many ways 30 marks can be allotted to 8 questions, if each question carries atleast 2 marks'. **[Manisha, Delhi]**

➥ Dear Manisha, for this problem you should know about the following concept.

'Number of non-negative integral solutions of the equation $x_1 + x_2 + \ldots + x_r = n$ or number of ways of distributing n identical objects among r groups when each group can get zero or more objects, is ${}^{n+r-1}C_{r-1}$.'

Now, let first, second, ..., eight questions be allotted $x_1, x_2, \ldots, x_8$ marks respectively, then we have

$x_1 + x_2 + \ldots + x_8 = 30$, where $x_1, x_2, \ldots, x_8 \geq 2$ ...(i)

Now, let $y_i = x_i - 2$, then

$$y_1 + y_2 + \ldots + y_8 = (x_1 + x_2 + \ldots + x_8) - 16$$

$\Rightarrow y_1 + y_2 + \ldots + y_8 = 30 - 16 = 14$, where y_i's ≥ 0 ...(ii)

Hence, required number

= Number of integral solutions of Eq. (i)

= Number of integral solutions of Eq. (ii)

$= {}^{14+8-1}C_{8-1}$ [$\because n = 14$ and $r = 8$]

$= {}^{21}C_7 = 116280$

Q3 Sir, I am unable to crack the following question. 'Let $f(x) = ax^2 + bx + c$, $a, b, c \in R$ and $a \neq 0$. Suppose $f(x) > 0$ for all $x \in R$ and $g(x) = f(x) + f'(x) + f''(x)$. Then,

(a) $g(x) > 0, \forall\ x \in R$ (b) $g(x) < 0, \forall\ x \in R$

(c) $g(x) = 0, \forall\ x \in R$ (d) $g(x) = 0$ has real roots

Please provide me its solution with detailed explanation. **[Abhishika, Kolkata]**

➥ Dear Abhishika, to crack this question you should know about a result stated below

'A quadratic expression $f(x) = ax^2 + bx + c > 0 (< 0), \forall\ x \in R$ iff $a > 0 (< 0)$ and $D = b^2 - 4ac < 0$.'

Here, we have $f(x) > 0$ for all $x \in R$

$\therefore\ a > 0$ and $b^2 - 4ac < 0$

Now, consider $g(x) = f(x) + f'(x) + f''(x)$

$= ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (2a + b)\,x + (2a + b + c)$

$\because\quad a > 0$ and $D = (2a + b)^2 - 4a\,(2a + b + c)$

$= 4a^2 + b^2 + 4ab - 8a^2 - 4ab - 4ac$

$= b^2 - 4ac - 4a^2 < 0$, as $b^2 - 4ac < 0$

$\therefore\ g(x) > 0,\ \forall\ x \in R$

Hence, option 'a' is correct.

Q4 Sir, I am facing problem in finding the solution of question given below. 'Find the equation of the circle of minimum radius which contains the three circles

$x^2 - y^2 - 4y - 5 = 0$

$x^2 + y^2 + 12x + 4y + 31 = 0$

$x^2 + y^2 + 6x + 12y + 36 = 0$'

Please provide me best solution. **[Abdul, Mumbai]**

➥ Dear Abdul, here is the best solution of your question.

Let $C(h, k)$ be the centre of the circle passing through the centres of three given circle.

Clearly, the coordinates of centres and radii of three given circles are

$C_1 \equiv (0, 2);\ r_1 = 3$

$C_2 \equiv (-6, -2);\ r_2 = 3$

$C_3 \equiv (-3, -6);\ r_3 = 3$

$\because\ CC_1 = CC_2 = CC_3$

$\therefore\ (CC_1)^2 = (CC_2)^2 = (CC_3)^2$

$\Rightarrow (h - 0)^2 + (k - 2)^2 = (h + 6)^2 + (k + 2)^2$

$= (h + 3)^2 + (k + 6)^2$

$\Rightarrow\quad h^2 + k^2 + 4 - 4k$

$= h^2 + 36 + 12h + k^2 + 4 + 4k$

$= h^2 + 9 + 6h + k^2 + 36 + 12k$

$\Rightarrow\quad -4k + 4 = 12h + 4k + 40 = 6h + 12k + 45$

$\Rightarrow\quad 12h + 8k + 36 = 0 \Rightarrow 3h + 2k + 9 = 0$ ...(i)

and $6h - 8k - 5 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$h = -\frac{31}{18}$ and $k = -\frac{23}{12}$

Now, $CP = CC_3 + C_3P$, where P is any point on the required circle.

$= CC_3 + 3 = \sqrt{\left(-3 + \frac{31}{18}\right)^2 + \left(-6 + \frac{23}{12}\right)^2} + 3$

$= \left(\frac{5}{36}\sqrt{949} + 3\right)$

Hence, equation of required circle is

$\left(x + \frac{31}{18}\right)^2 + \left(y + \frac{23}{12}\right)^2 = \left(\frac{5}{36}\sqrt{949} + 3\right)^2$

Q5 Sir, please provide me a shortcut to find the solution of the following problem

'If $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2\left(\frac{x}{2}\right) \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$, then find the value of $\int_{-\pi/2}^{\pi/2} (x^2 + 1)\{f(x) + f''(x)\}\,dx$.'

[Suraj, Kanpur]

➥ Dear Suraj, to find the solution of above problem you should know about the concept of even-odd function and properties of definite integral, which are discussed below

(i) If $f(-x) = f(x)$, then f is even function

(ii) If $f(-x) = -f(x)$, then f is odd function

(iii) $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f \text{ is even function} \\ 0, & \text{if } f \text{ is odd function} \end{cases}$

Consider, $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2\left(\frac{x}{2}\right) \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$

As $f(-x) = -f(x)$

$\therefore\quad f(x)$ is odd function

$\Rightarrow\quad f'(x)$ is even function

$\Rightarrow\quad f''(x)$ is odd function

Now, consider $\phi(x) = (x^2 + 1)\{f(x) + f''(x)\}$

$\Rightarrow\ \phi(-x) = -\phi(x)$, i.e. $\phi(x)$ is an odd function.

Hence, $\int_{-\pi/2}^{\pi/2} \phi(x)\,dx = 0$.

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Questions to Measure Your Problem Solving Skills

Instructions

- This test consists of 45 questions and each question is allotted 3 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. For all $n \in N$, $2^{4n} - 15n - 1$ is divisible by

(a) 225 (b) 125
(c) 325 (d) None of these

2. If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$ then the value of $\int_0^{\pi/2} f(x)\,dx$ is

(a) 3 (b) 0 (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

3. If the third term in the expansion of $\left(\frac{1}{x} + x^{\log_{10} x}\right)^5$, $x > 1$ is 1000, then the value of x is

(a) $\sqrt{10}$ (b) 10
(c) 100 (d) $10\sqrt{10}$

4. The equation of the plane which contains two parallel lines $\frac{x+1}{3} = \frac{y-2}{2} = \frac{z}{1}$ and $\frac{x-3}{3} = \frac{y+4}{2} = \frac{z-1}{1}$, is

(a) $3x + 2y + z = 14$ (b) $8x + y - 26z + 6 = 0$
(c) $4x - 6y + 2 = 53$ (d) None of these

5. If the lines $4x + 3y - 1 = 0$, $x - y + 5 = 0$ and $kx + 5y - 3 = 0$ are concurrent, then k is equal to

(a) 4 (b) 5 (c) 6 (d) 7

6. The differential equation $(e^x + 1)y\,dy = (y + 1)\,e^x\,dx$ has the solution

(a) $(y - 1)(e^x - 1) = Ce^y$ (b) $(y - 1)(e^x + 1) = Ce^y$
(c) $(y + 1)(e^x - 1) = Ce^y$ (d) $(y + 1)(e^x + 1) = Ce^y$

7. The value of $n \in I$ for which the function $f(x) = \frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ has 4π as its period, is

(a) 2 (b) 3 (c) 4 (d) 5

8. The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the X-axis, is

(a) $\sqrt{3}y = 3x + 1$ (b) $\sqrt{3}\,y = -(x + 3)$
(c) $\sqrt{3}\,y = x + 3$ (d) $\sqrt{3}y = -(3x + 1)$

9. $\int \left(\frac{1}{1 - x^4}\right)\left[\cos^{-1}\frac{2x}{1 + x^2} + \tan^{-1}\frac{2x}{1 - x^2}\right] dx$ is equal to

(a) $\frac{\pi}{4}\left[\tan^{-1}x + \frac{1}{2}\log\left|\frac{1 - x}{1 + x}\right|\right] + C$

(b) $\frac{\pi}{4}\left[\tan^{-1}x - \frac{1}{2}\log\left|\frac{1 - x}{1 + x}\right|\right] + C$

(c) $\frac{\pi}{4}\left[\tan^{-1}x - \frac{1}{2}\log\left|\frac{1 + x}{1 - x}\right|\right] + C$

(d) None of the above

10. A telephone company in a town has 500 subscribers on its list and collects fixed charges of ₹ 300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of ₹ 1, one subscriber will discontinue the service. What increase will bring maximum income to the company?

(a) ₹ 50 (b) ₹ 25
(c) ₹ 100 (d) ₹ 150

11. Let $f(x) = \begin{cases} \frac{1+\cos x}{(\pi - x)^2} \cdot \frac{\sin^2 x}{\log(1+\pi^2 - 2\pi x + x^2)}, & x \neq \pi \\ k, & x = \pi \end{cases}$

If $f(x)$ is continuous at $x = \pi$, then k is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{-1}{2}$ (d) $\frac{-1}{4}$

12. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then $1 + \frac{c}{a} + \left|\frac{b}{a}\right|$ is

(a) < 0 (b) > 0

(c) ≤ 0 (d) None of these

13. The total number of tangents through the points (3, 5) that can be drawn to the ellipse $3x^2 + 5y^2 = 32$ and $25x^2 + 9y^2 = 450$ is

(a) 0 (b) 2 (c) 3 (d) 4

14. The value of $\lim_{x \to 0} (\cos x)^{\cot^2 x}$ is

(a) e^{-1} (b) $e^{-1/2}$

(c) 1 (d) Does not exist

15. If ω is a root of unity and $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$, then A^{-1} is equal to

(a) $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$ (d) $\frac{1}{2}\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$

16. Let a, b, c be in AP and $|a| < 1, |b| < 1, |c| < 1$.
If $x = 1 + a + a^2 + \ldots \infty$, $y = 1 + b + b^2 + \ldots \infty$, $z = 1 + c + c^2 + \ldots \infty$, then x^{-1}, y^{-1}, z^{-1} are in

(a) AP (b) GP

(c) HP (d) None of these

17. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then the least integral value of m is

(a) 2 (b) 4

(c) 8 (d) None of these

18. $\lim_{x \to \infty} \frac{\int_0^{2x} xe^{x^2} dx}{e^{4x^2}}$ is equal to

(a) 0 (b) ∞

(c) 2 (d) $\frac{1}{2}$

19. $\log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + \ldots$ is equal to

(a) $(\log_e 3)(\log_e 2)$ (b) $\log_e 3$

(c) $\log_e 2$ (d) $\frac{\log_e 5}{\log_e 3}$

20. The AM of n observations is M. If the sum of $n - 4$ observations is a, then the mean of remaining 4 observations is

(a) $\frac{nM - a}{4}$ (b) $\frac{nM - a}{2}$

(c) $\frac{nM + a}{4}$ (d) $nM + a$

21. If $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

22. A box contains 24 balls of which 12 are black and 12 are white. The balls are drawn from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is

(a) $\frac{5}{64}$ (b) $\frac{27}{32}$ (c) $\frac{5}{32}$ (d) $\frac{11}{32}$

23. ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$, if n

(a) > 5 (b) > 6

(c) > 7 (d) None of these

24. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4 and then the men select the chairs from amongst the remaining. The number of possible arrangements is

(a) ${}^4C_3 \times {}^4C_2$ (b) ${}^4C_2 \times {}^4P_3$

(c) ${}^4P_2 \times {}^4P_3$ (d) None of these

25. The sum of the coefficient of each power of x in the expansion of $(1 + x + x^2 + x^3)^5$ is

(a) 256 (b) 128

(c) 512 (d) 1024

26. The distance of the point (3, 8, 2) from the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ measured parallel to the plane $3x + 2y - 2z = 0$ is

(a) 2 (b) 3 (c) 6 (d) 7

27. If x, y and z are complex numbers, then $\Delta = \begin{vmatrix} 0 & -y & -z \\ \bar{y} & 0 & -x \\ \bar{z} & \bar{x} & 0 \end{vmatrix}$ is

(a) purely real (b) purely imaginary

(c) complex (d) 0

28. The area enclosed between the curves $y = x^3$ and $y = \sqrt{x}$ is

(a) $\frac{5}{3}$ sq units (b) $\frac{5}{4}$ sq units

(c) $\frac{5}{12}$ sq unit (d) $\frac{12}{5}$ sq units

29. The number of real roots of the equation $\left(x+\frac{1}{x}\right)^3 + x + \frac{1}{x} = 0$ is

(a) 0 (b) 2 (c) 4 (d) 6

30. If the vertices of a triangle have integral coordinates, then the triangle is

(a) equilateral (b) never equilateral
(c) isosceles (d) None of these

31. The Rolle's theorem is applicable in the interval $-1 \le x \le 1$. For the function

(a) $f(x) = x$ (b) $f(x) = x^2$
(c) $f(x) = 2x^3 + 3$ (d) $f(x) = |x|$

32. The absolute value of $\int_{10}^{19} \frac{\sin x}{1+x^8}\,dx$ is

(a) less than 10^{-7} (b) more than 10^{-7}
(c) less than 10^{-9} (d) None of these

33. $1+\frac{2^3}{2!}+\frac{3^3}{3!}+\frac{4^3}{4!}+\ldots$ is equal to

(a) $5e$ (b) $4e$ (c) $3e$ (d) $2e$

34. If the vectors $\mathbf{a} = (c \log_2 x)\,\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{b} = (\log_2 x)\,\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + (2c \log_2 x)\,\hat{\mathbf{k}}$ make an obtuse angle for any $x \in (0, \infty)$, then the interval of which c belongs

(a) $\left(\frac{-4}{3}, 0\right)$ (b) $\left(\infty, \frac{-4}{3}\right)$
(c) $\left(\frac{3}{4}, 0\right)$ (d) $\left(\frac{-3}{4}, 0\right)$

35. The function $f(x) = x^2 e^{-x}$ increases in the interval

(a) (0, 2) (b) (2, 3)
(c) (3, 4) (d) (4, 5)

36. The average weight of students in a class of 35 students is 40 kg. If the weight of the teacher be included, the average rises by $\frac{1}{2}$ kg, then the weight of the teacher is

(a) 40.5 kg (b) 50 kg
(c) 41 kg (d) 58 kg

37. The straight lines $3x + 4y - 5 = 0$ and $4x = 3y + 15$ intersect at the point P. On these lines the points Q and R are chosen so that $PQ = PR$. The slopes of the lines QR passing through (1, 2) are

(a) $-7, \frac{1}{7}$ (b) $7, \frac{1}{7}$
(c) $7, \frac{-1}{7}$ (d) $3, \frac{-1}{3}$

38. Which of the following is not true for any two statements p and q?

(a) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge q$
(b) $(p \vee q) \vee (\sim q)$ is a tautology
(c) $(p \wedge q) \wedge (\sim q)$ is contradiction
(d) $\sim(p \vee q) \equiv (\sim p) \vee (\sim q)$

39. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. Atmost 19 passed Mathematics and Physics, atmost 29 Mathematics and Chemistry and atmost 20 Physics and Chemistry. The largest possible number that could have passed all three exam, is

(a) 10 (b) 12
(c) 9 (d) None of these

40. The greatest positive integer, which divides $(n+2)(n+3)(n+4)(n+5)(n+6)$ for all $n \in N$, is

(a) 4 (b) 120 (c) 240 (d) 24

41. If the given constraints are $5x + 4y \ge 2$, $x \le 6$, $y \le 7$ and $x, y \ge 0$, then the maximum value of the function $Z = x + 2y$ is

(a) 13 (b) 14 (c) 15 (d) 20

42. The differential equation of the family of circles passing through the fixed point $(a, 0)$ and $(-a, 0)$ is

(a) $y'(y^2 - x^2) + 2xy + a^2 = 0$
(b) $y'y^2 + xy + a^2x^2 = 0$
(c) $y'(y^2 - x^2 + a^2) + 2xy = 0$
(d) $y'(y^2 + x^2) - 2xy + a^2 = 0$

43. If c, d are roots of $x^2 - 10ax - 11b = 0$ and a, b are roots of $x^2 - 10cx - 11d = 0$, then value of $a + b + c + d$ is

(a) 1210 (b) – 1 (c) 2530 (d) – 11

44. If $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ for all $x \in R - \{0\}$, then $f(x^4)$ is equal to

(a) $\frac{(1-x^4)(2x^4+3)}{5x^4}$ (b) $\frac{(1+x^4)(2x^4-3)}{5x^4}$
(c) $\frac{(1-x^4)(2x^4-3)}{5x^4}$ (d) None of these

45. In a club election, the number of contestants is one more than the number of maximum candidate for which a voter can vote. If the total number of ways in which a voter can vote is 126, then the number of contestants is

(a) 4 (b) 5 (c) 6 (d) 7

Answers

1. (a)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (c)
11. (b)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (a)
21. (a)	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (b)	**28.** (c)	**29.** (a)	**30.** (b)
31. (b)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (d)	**37.** (a)	**38.** (d)	**39.** (d)	**40.** (b)
41. (d)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (d)					

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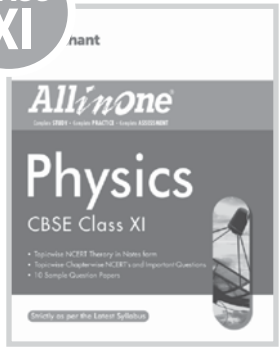
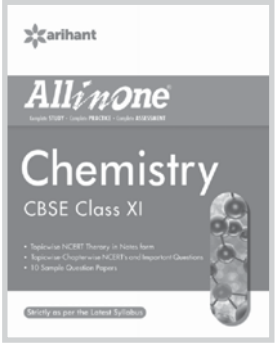


F246 ₹450



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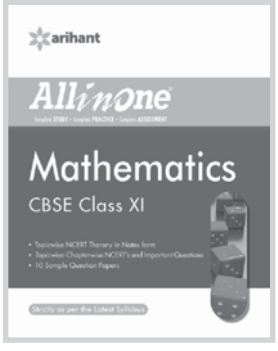

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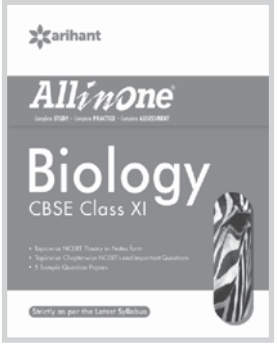

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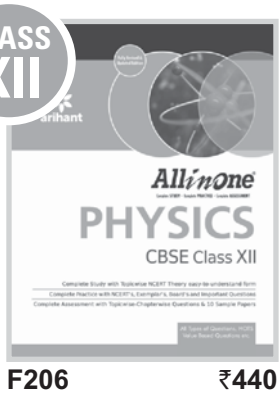

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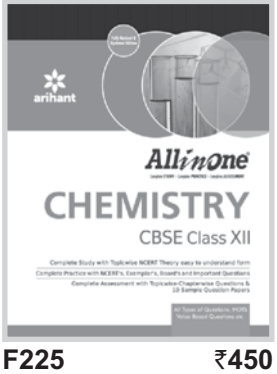

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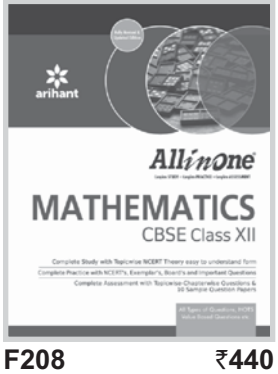

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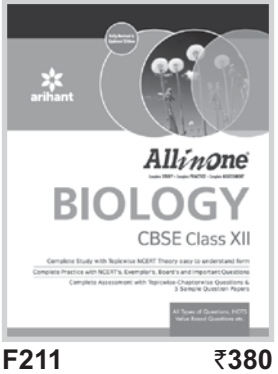

F211 ₹380

AVAILABLE IN

Physics, Chemistry, Mathematics, Biology, Accountancy, Business Studies, Economics, Computer Science, Informatics Practices, English Core, Hindi Core

arihant

Exam held on 3 April, 2016

{EXAM DIARY}

JEE Main 2016 (OFFLINE) SOLVED PAPER

General Instructions

- There are three parts in the question paper *A*, *B*, *C* consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
- Each question is allotted 4 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made, if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

EXAM CRUX

This year Mathematics question paper was quite balanced as 20% questions were HOTS, 45% questions were moderate and 35% questions were easy. Overall, it was easier as compared to previous year paper. In classwise division of questions it is found that 55% questions are asked from class-XI and 45% questions are asked from class-XII.

According to branchwise division, it is found that 5 questions are asked from Coordinate Geometry; 2 questions are asked from 3D-Geometry; 9 questions are asked from Calculus, 10 questions are asked from Algebra; 2 questions are asked from Trigonometry, 1 question is asked from Statistics and 1 question is asked from vector.

1. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S

(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements

2. A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary, is

(a) $\frac{\pi}{3}$ *(b)* $\frac{\pi}{6}$
(c) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ *(d)* $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

3. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is

(a) 3 *(b)* – 4 *(c)* 6 *(d)* 5

4. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,\text{adj}\,A = AA^T$, then $5a + b$ is equal to

(a) – 1 *(b)* 5 *(c)* 4 *(d)* 13

5. The system of linear equations
$x + \lambda y - z = 0;\ \lambda x - y - z = 0;\ x + y - \lambda z = 0$
has a non-trivial solution for

(a) infinitely many values of λ
(b) exactly one value of λ
(c) exactly two values of λ
(d) exactly three values of λ

6. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is

(a) 46th *(b)* 59th *(c)* 52nd *(d)* 58th

7. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n,\ x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is

(a) 64 *(b)* 2187 *(c)* 243 *(d)* 729

8. If the 2nd, 5th and 9th terms of a non-constant AP are in GP, then the common ratio of this GP is

(a) $\frac{8}{5}$ *(b)* $\frac{4}{3}$
(c) 1 *(d)* $\frac{7}{4}$

9. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots$, is $\frac{16}{5}m$, then m is equal to

(a) 102 (b) 101
(c) 100 (d) 99

10. Let $p = \lim\limits_{x \to 0^+} (1 + \tan^2 \sqrt{x})^{1/2x}$, then $\log p$ is equal to

(a) 2 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

11. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

(a) g is not differentiable at $x = 0$
(b) $g'(0) = \cos(\log 2)$
(c) $g'(0) = -\cos(\log 2)$
(d) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

12. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$.

A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point

(a) (0, 0) (b) $\left(0, \frac{2\pi}{3}\right)$
(c) $\left(\frac{\pi}{6}, 0\right)$ (d) $\left(\frac{\pi}{4}, 0\right)$

13. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then

(a) $2x = (\pi + 4)r$ (b) $(4 - \pi)x = \pi r$
(c) $x = 2r$ (d) $2x = r$

14. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}\,dx$ is equal to

(a) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$

(b) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$

(c) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$

(d) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$

where, C is an arbitrary constant.

15. $\lim\limits_{n \to \infty} \left(\frac{(n+1)(n+2)\ldots 3n}{n^{2n}}\right)^{1/n}$ is equal to

(a) $\frac{18}{e^4}$ (b) $\frac{27}{e^2}$
(c) $\frac{9}{e^2}$ (d) $3\log 3 - 2$

16. The area (in sq units) of the region $\{(x, y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is

(a) $\pi - \frac{4}{3}$ (b) $\pi - \frac{8}{3}$
(c) $\pi - \frac{4\sqrt{2}}{3}$ (d) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

17. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to

(a) $-\frac{2}{5}$ (b) $-\frac{4}{5}$
(c) $\frac{2}{5}$ (d) $\frac{4}{5}$

18. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?

(a) $(-3, -9)$ (b) $(-3, -8)$
(c) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (d) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

19. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the X-axis, lie on

(a) a circle
(b) an ellipse which is not a circle
(c) a hyperbola
(d) a parabola

20. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3, 2)$, then the radius of S is

(a) $5\sqrt{2}$ (b) $5\sqrt{3}$
(c) 5 (d) 10

21. Let P be the point on the parabola, $y^2 = 8x$, which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then, the equation of the circle, passing through C and having its centre at P is

(a) $x^2 + y^2 - 4x + 8y + 12 = 0$
(b) $x^2 + y^2 - x + 4y - 12 = 0$
(c) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
(d) $x^2 + y^2 - 4x + 9y + 18 = 0$

22. The eccentricity of the hyperbola whose length of the latusrectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$
(c) $\frac{2}{\sqrt{3}}$ (d) $\sqrt{3}$

23. The distance of the point (1, – 5, 9) from the plane x – y + z = 5 measured along the line x = y = z is

(a) $3\sqrt{10}$ (b) $10\sqrt{3}$

(c) $\frac{10}{\sqrt{3}}$ (d) $\frac{20}{3}$

24. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to

(a) 26 (b) 18

(c) 5 (d) 2

25. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three unit vectors such that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\frac{\sqrt{3}}{2}(\mathbf{b}+\mathbf{c})$. If $\mathbf{b}$ is not parallel to $\mathbf{c}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

26. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(a) $3a^2 - 26a + 55 = 0$

(b) $3a^2 - 32a + 84 = 0$

(c) $3a^2 - 34a + 91 = 0$

(d) $3a^2 - 23a + 44 = 0$

27. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is not true?

(a) E_1 and E_2 are independent

(b) E_2 and E_3 are independent

(c) E_1 and E_3 are independent

(d) E_1, E_2 and E_3 are independent

28. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation

$$\cos x + \cos 2x + \cos 3x + \cos 4x = 0, \text{ is}$$

(a) 3 (b) 5

(c) 7 (d) 9

29. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 min from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then, the time taken (in minutes) by him, from B to reach the pillar, is

(a) 6 (b) 10 (c) 20 (d) 5

30. The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to

(a) $\sim p \wedge q$ (b) $p \wedge q$

(c) $p \vee q$ (d) $p \vee \sim q$

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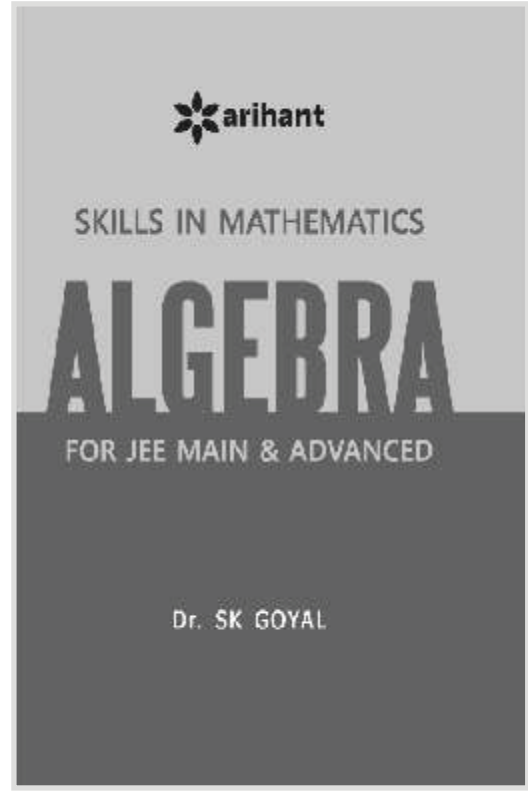


B011 **₹635**

Dr. SK GOYAL

Answers *with* Explanation

1. ***(c)*** We have, $f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \neq 0$ ...(i)

On replacing x by $\frac{1}{x}$ in the above equation, we get

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$$

$$\Rightarrow \quad 2f(x) + f\left(\frac{1}{x}\right) = \frac{3}{x} \qquad \text{...(ii)}$$

On multiplying Eq. (ii) by 2 and subtracting Eq. (i) from Eq. (ii), we get

$$4f(x) + 2f\left(\frac{1}{x}\right) = \frac{6}{x}$$

$$\underline{f(x) + 2f\left(\frac{1}{x}\right) = 3x}$$

$$3f(x) = \frac{6}{x} - 3x$$

$$\Rightarrow \quad f(x) = \frac{2}{x} - x$$

Now, consider $f(x) = f(-x)$

$$\Rightarrow \quad \frac{2}{x} - x = -\frac{2}{x} + x$$

$$\Rightarrow \quad \frac{4}{x} = 2x$$

$$\Rightarrow \quad 2x^2 = 4$$

$$\Rightarrow \quad x^2 = 2$$

$$\Rightarrow \quad x = \pm\sqrt{2}$$

Hence, S contains exactly two elements.

2. ***(d)*** Let $z = \frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary. Then, we have $\text{Re}(z) = 0$.

Now consider, $z = \frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$

$$= \frac{(2 + 3i\sin\theta)(1 + 2i\sin\theta)}{(1 - 2i\sin\theta)(1 + 2i\sin\theta)}$$

$$= \frac{2 + 4i\sin\theta + 3i\sin\theta + 6i^2\sin^2\theta}{1^2 - (2i\sin\theta)^2}$$

$$= \frac{2 + 7i\sin\theta - 6\sin^2\theta}{1 + 4\sin^2\theta}$$

$$= \frac{2 - 6\sin^2\theta}{1 + 4\sin^2\theta} + i\,\frac{7\sin\theta}{1 + 4\sin^2\theta}$$

$\because \quad \text{Re}(z) = 0$

$\therefore \quad \frac{2 - 6\sin^2\theta}{1 + 4\sin^2\theta} = 0$

$$\Rightarrow \quad 2 = 6\sin^2\theta$$

$$\Rightarrow \quad \sin^2\theta = \frac{1}{3}$$

$$\Rightarrow \quad \sin\theta = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\pm\frac{1}{\sqrt{3}}\right) = \pm\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

3. ***(a)*** Given, $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$

Clearly, this is possible when

I. $x^2 + 4x - 60 = 0$ and $x^2 - 5x + 5 \neq 0$

or

II. $x^2 - 5x + 5 = 1$

or

III. $x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60 =$ even integer.

Case I When $x^2 + 4x - 60 = 0$

$$\Rightarrow \quad x^2 + 10x - 6x - 60 = 0$$

$$\Rightarrow \quad x(x + 10) - 6(x + 10) = 0$$

$$\Rightarrow \quad (x + 10)(x - 6) = 0$$

$$\Rightarrow \quad x = -10 \text{ or } x = 6$$

Note that for these two values of x, $x^2 - 5x + 5 \neq 0$

Case II When $x^2 - 5x + 5 = 1$

$$\Rightarrow \quad x^2 - 5x + 4 = 0$$

$$\Rightarrow \quad x^2 - 4x - x + 4 = 0$$

$$\Rightarrow \quad x(x - 4) - 1(x - 4) = 0$$

$$\Rightarrow \quad (x - 4)(x - 1) = 0$$

$$\Rightarrow \quad x = 4 \text{ or } x = 1$$

Case III When $x^2 - 5x + 5 = -1$

$$\Rightarrow \quad x^2 - 5x + 6 = 0$$

$$\Rightarrow \quad x^2 - 2x - 3x + 6 = 0$$

$$\Rightarrow \quad x(x - 2) - 3(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)(x - 3) = 0$$

$$\Rightarrow \quad x = 2 \text{ or } x = 3$$

Now, when $x = 2$, $x^2 + 4x - 60 = 4 + 8 - 60 = -48$, which is an even integer.

When $x = 3$, $x^2 + 4x - 60 = 9 + 12 - 60 = -39$, which is not an even integer.

Thus, in this case, we get $x = 2$.

Hence, sum of all real values of $x = -10 + 6 + 4 + 1 + 2 = 3$

4. ***(b)*** Given, $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj $A = AA^T$

Clearly, $A(\text{adj } A) = |A|\, I_2$ [$\because$ if A is square matrix of order n, then $A(\text{adj } A) = (\text{adj } A)\cdot A = |A|\, I_n$]

$$= \begin{vmatrix} 5a & -b \\ 3 & 2 \end{vmatrix} I_2$$

$$= (10a + 3b)\, I_2 = (10a + 3b)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix} \qquad \text{...(i)}$$

and $$AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} \qquad \text{...(ii)}$$

$\because \quad A(\text{adj } A) = AA^T$

$\therefore \quad \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix} = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

[using Eqs. (i) and (ii)]

$\Rightarrow \quad 15a - 2b = 0 \Rightarrow a = \frac{2b}{15}$ ...(iii)

and $10a + 3b = 13$ ...(iv)

On substituting the value of 'a' from Eq. (iii) in Eq. (iv), we get

$$10 \cdot \left(\frac{2b}{15}\right) + 3b = 13 \Rightarrow \frac{20b + 45b}{15} = 13 \Rightarrow \frac{65b}{15} = 13$$

$\Rightarrow \quad b = 3$

Now, on substituting the value of b in Eq. (iii), we get $5a = 2$

Hence, $5a + b = 2 + 3 = 5$

5. ***(d)*** Given, system of linear equation is

$x + \lambda y - z = 0; \quad \lambda x - y - z = 0; \quad x + y - \lambda z = 0$

Note that, given system will have a non-trivial solution only if determinant of coefficient matrix is zero, i.e. $\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow \quad 1(\lambda + 1) - \lambda(-\lambda^2 + 1) - 1(\lambda + 1) = 0$

$\Rightarrow \quad \lambda + 1 + \lambda^3 - \lambda - \lambda - 1 = 0$

$\Rightarrow \quad \lambda^3 - \lambda = 0$

$\Rightarrow \quad \lambda(\lambda^2 - 1) = 0$

$\Rightarrow \quad \lambda = 0 \text{ or } \lambda = \pm 1$

Hence, given system of linear equation has a non-trivial solution for exactly three values of λ.

6. ***(d)*** Clearly, number of words start with A $= \frac{4!}{2!} = 12$

Number of words start with L $= 4! = 24$

Number of words start with M $= \frac{4!}{2!} = 12$

Number of words start with SA $= \frac{3!}{2!} = 3$

Number of words start with SL $= 3! = 6$

Note that, next word will be "SMALL".

Hence, position of word "SMALL" is 58th.

7. ***(d)*** Clearly, number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$ is $\frac{(n+2)(n+1)}{2}$ or ${}^{n+2}C_2$.

[assuming $\frac{1}{x}$ and $\frac{1}{x^2}$ distinct]

$\therefore \quad \frac{(n+2)(n+1)}{2} = 28$

$\Rightarrow \quad (n+2)(n+1) = 56 = (6+1)(6+2) \Rightarrow n = 6$

Hence, sum of coefficients $= (1 - 2 + 4)^6 = 3^6 = 729$

Note As $\frac{1}{x}$ and $\frac{1}{x^2}$ are functions of same variable, therefore number of dissimilar terms will be $2n + 1$, i.e. odd, which is not possible. Hence, it contains error.

8. ***(b)*** Let a be the first term and d be the common difference. Then, we have $a + d, a + 4d, a + 8d$ in GP

i.e. $\quad (a + 4d)^2 = (a + d)(a + 8d)$

$\Rightarrow \quad a^2 + 16d^2 + 8ad = a^2 + 8ad + ad + 8d^2$

$\Rightarrow \quad 8d^2 = ad$

$\Rightarrow \quad 8d = a \quad [\because d \neq 0]$

Now, common ratio, $(r) = \frac{a + 4d}{a + d} = \frac{8d + 4d}{8d + d} = \frac{12d}{9d} = \frac{4}{3}$

9. ***(b)*** Let S_{10} be the sum of first ten terms of the series. Then, we have

$$S_{10} = \left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots \text{ to 10 terms}$$

$$= \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + 4^2 + \left(\frac{24}{5}\right)^2 + \ldots \text{ to 10 terms}$$

$$= \frac{1}{5^2}(8^2 + 12^2 + 16^2 + 20^2 + 24^2 + \ldots \text{ to 10 terms})$$

$$= \frac{4^2}{5^2}(2^2 + 3^2 + 4^2 + 5^2 + \ldots \text{ to 10 terms})$$

$$= \frac{4^2}{5^2}(2^2 + 3^2 + 4^2 + 5^2 + \ldots + 11^2)$$

$$= \frac{16}{25}((1^2 + 2^2 + \ldots + 11^2) - 1^2)$$

$$= \frac{16}{25}\left(\frac{11 \cdot (11+1)(2 \cdot 11 + 1)}{6} - 1\right)$$

$$= \frac{16}{25}(506 - 1) = \frac{16}{25} \times 505$$

$\Rightarrow \frac{16}{5}m = \frac{16}{25} \times 505$

$\Rightarrow \quad m = 101$

10. ***(c)*** Given, $p = \lim_{x \to 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ (1^∞ form)

$$= e^{\lim_{x \to 0^+} \frac{\tan^2 \sqrt{x}}{2x}} = e^{\frac{1}{2}\lim_{x \to 0^+} \left(\frac{\tan \sqrt{x}}{\sqrt{x}}\right)^2} = e^{\frac{1}{2}}$$

$\therefore \quad \log p = \log e^{\frac{1}{2}} = \frac{1}{2}$

11. ***(b)*** We have, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x)), x \in R$

Note that, for $x \to 0, \log 2 > \sin x$

$\therefore \quad f(x) = \log 2 - \sin x$

$\Rightarrow \quad g(x) = \log 2 - \sin(f(x))$

$= \log 2 - \sin(\log 2 - \sin x)$

Clearly, $g(x)$ is differentiable at $x = 0$ as $\sin x$ is differentiable.

Now, $\quad g'(x) = -\cos(\log 2 - \sin x)(-\cos x)$

$= \cos x \cdot \cos(\log 2 - \sin x)$

$\Rightarrow \quad g'(0) = 1 \cdot \cos(\log 2)$

12. ***(b)*** We have, $f(x) = \tan^{-1}\sqrt{\frac{1 + \sin x}{1 - \sin x}}, x \in \left(0, \frac{\pi}{2}\right)$

$$\Rightarrow \quad f(x) = \tan^{-1}\sqrt{\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}$$

$$= \tan^{-1}\left(\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right)$$

$\left[\because \cos\frac{x}{2} > \sin\frac{x}{2} \text{ for } 0 < \frac{x}{2} < \frac{\pi}{4}\right]$

$$= \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

$$= \frac{\pi}{4}+\frac{x}{2}$$

$$\Rightarrow \quad f'(x) = \frac{1}{2} \Rightarrow f'\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

Now, equation of normal at $x = \frac{\pi}{6}$ is given by

$$\left(y - f\left(\frac{\pi}{6}\right)\right) = -2\left(x - \frac{\pi}{6}\right)$$

$$\Rightarrow \quad \left(y - \frac{\pi}{3}\right) = -2\left(x - \frac{\pi}{6}\right)$$

$$\left[\because f\left(\frac{\pi}{6}\right) = \frac{\pi}{4}+\frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}\right]$$

which passes through $\left(0, \frac{2\pi}{3}\right)$.

13. ***(c)*** According to given information, we have

Perimeter of square + Perimeter of circle = 2 units

$$\Rightarrow \quad 4x + 2\pi r = 2$$

$$\Rightarrow \quad r = \frac{1-2x}{\pi} \qquad \ldots(i)$$

Now, let A be the sum of the areas of the square and the circle. Then,

$$A = x^2 + \pi r^2$$

$$= x^2 + \pi\,\frac{(1-2x)^2}{\pi^2}$$

$$\Rightarrow \quad A(x) = x^2 + \frac{(1-2x)^2}{\pi}$$

Now, for minimum value of $A(x)$, $\frac{dA}{dx} = 0$

$$\Rightarrow \quad 2x + \frac{2(1-2x)}{\pi}\cdot(-2) = 0$$

$$\Rightarrow \quad x = \frac{2-4x}{\pi}$$

$$\Rightarrow \quad \pi x + 4x = 2$$

$$\Rightarrow \quad x = \frac{2}{\pi+4} \qquad \ldots(ii)$$

Now, from Eq. (i), we get

$$r = \frac{1-2\cdot\frac{2}{\pi+4}}{\pi}$$

$$= \frac{\pi+4-4}{\pi(\pi+4)} = \frac{1}{\pi+4} \qquad \ldots(iii)$$

From Eqs. (ii) and (iii), we get $x = 2r$

14. ***(b)*** Let $I = \int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}\,dx$

$$= \int \frac{2x^{12}+5x^9}{x^{15}(1+x^{-2}+x^{-5})^3}\,dx$$

$$= \int \frac{5x^{-6}+2x^{-3}}{(1+x^{-2}+x^{-5})^3}\,dx$$

Now, put $1 + x^{-2} + x^{-5} = t$

$$\Rightarrow \quad (-2x^{-3} - 5x^{-6})\,dx = dt$$

$$\Rightarrow \quad (2x^{-3} + 5x^{-6})\,dx = -dt$$

$$\therefore \quad I = -\int \frac{dt}{t^3} = -\int t^{-3}\,dt$$

$$= -\frac{t^{-3+1}}{-3+1} + C = \frac{1}{2t^2} + C$$

$$= \frac{x^{10}}{2(x^5+x^3+1)^2} + C$$

15. ***(b)*** Let $I = \lim_{n\to\infty}\left(\frac{(n+1)\cdot(n+2)\ldots(3n)}{n^{2n}}\right)^{\frac{1}{n}}$

$$= \lim_{n\to\infty}\left(\frac{(n+1)\cdot(n+2)\ldots(n+2n)}{n^{2n}}\right)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n}\right)\ldots\left(\frac{n+2n}{n}\right)\right)^{\frac{1}{n}}$$

On taking log both sides, we get

$$\log I = \lim_{n\to\infty}\frac{1}{n}\left[\log\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{2n}{n}\right)\right\}\right]$$

$$\Rightarrow \log I = \lim_{n\to\infty}\frac{1}{n}\left[\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+\ldots+\log\left(1+\frac{2n}{n}\right)\right]$$

$$\Rightarrow \log I = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\frac{r}{n}\right)$$

$$\Rightarrow \log I = \int_0^2 \log(1+x)\,dx$$

$$\Rightarrow \log I = \left[\log(1+x)\cdot x - \int\frac{1}{1+x}\cdot x\,dx\right]_0^2$$

$$\Rightarrow \log I = [\log(1+x)\cdot x]_0^2 - \int_0^2 \frac{x+1-1}{1+x}\,dx$$

$$\Rightarrow \log I = 2\cdot\log 3 - \int_0^2\left(1-\frac{1}{1+x}\right)dx$$

$$\Rightarrow \log I = 2\cdot\log 3 - [x - \log|1+x|]_0^2$$

$$\Rightarrow \log I = 2\cdot\log 3 - [2 - \log 3]$$

$$\Rightarrow \log I = 3\cdot\log 3 - 2$$

$$\Rightarrow \log I = \log 27 - 2$$

$$\Rightarrow \quad I = e^{\log 27 - 2}$$

$$= 27\cdot e^{-2} = \frac{27}{e^2}$$

16. ***(b)*** Given equations of curves are

$y^2 = 2x$, which is a parabola with vertex (0, 0) and axis parallel to X-axis. ...(i)

and $x^2 + y^2 = 4x$, which is a circle with centre (2, 0) and radius = 2 ...(ii)

On substituting $y^2 = 2x$ in Eq. (ii), we get

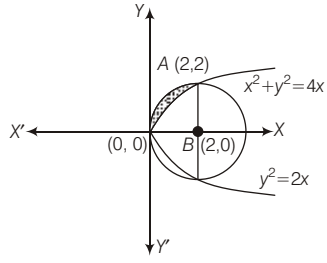
$$x^2 + 2x = 4x$$

$\Rightarrow$ $x^2 = 2x$

$\Rightarrow$ $x = 0$ or $x = 2$

$\Rightarrow$ $y = 0$ or $y = \pm 2$ [using Eq. (i)]

Now, the required area is the area of shaded region, i.e.



$$\text{Required area} = \frac{\text{Area of circle}}{4} - \int_0^2 \sqrt{2x}\, dx$$

$$= \frac{\pi(2)^2}{4} - \sqrt{2}\int_0^2 x^{1/2} dx$$

$$= \pi - \sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 = \pi - \frac{2\sqrt{2}}{3}[2\sqrt{2} - 0]$$

$$= \left(\pi - \frac{8}{3}\right) \text{sq. units}$$

17. ***(d)*** Given differential equation is

$$y(1 + xy)dx = x\, dy$$

$\Rightarrow$ $y\, dx + xy^2 dx = x\, dy$

$\Rightarrow$ $\frac{x\, dy - y\, dx}{y^2} = x\, dx$

$\Rightarrow$ $-\frac{(y\, dx - x\, dy)}{y^2} = x\, dx$

$\Rightarrow$ $-d\left(\frac{x}{y}\right) = x\, dx$

On integrating both sides, we get

$$-\frac{x}{y} = \frac{x^2}{2} + C$$

$\because$ It passes through $(1, -1)$.

$\therefore$ $1 = \frac{1}{2} + C$

$\Rightarrow$ $C = \frac{1}{2}$

Now, $-\frac{x}{y} = \frac{x^2}{2} + \frac{1}{2}$

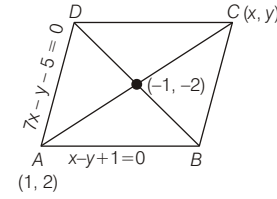
$\Rightarrow$ $x^2 + 1 = -\frac{2x}{y}$

$\Rightarrow$ $y = -\frac{2x}{x^2 + 1}$

$\Rightarrow$ $f\left(-\frac{1}{2}\right) = \frac{4}{5}$

18. ***(c)*** As the given lines $x - y + 1 = 0$ and $7x - y - 5 = 0$ are not parallel, therefore they represent the adjacent sides of the rhombus.

On solving $x - y + 1 = 0$ and $7x - y - 5 = 0$, we get $x = 1$ and $y = 2$. Thus, one of the vertex is $A(1, 2)$.



Let the coordinate of point C be (x, y). Then,

$$-1 = \frac{x + 1}{2} \text{ and } -2 = \frac{y + 2}{2}$$

$\Rightarrow$ $x + 1 = -2$ and $y = -4 - 2$

$\Rightarrow$ $x = -3$ and $y = -6$

Hence, coordinates of $C = (-3, -6)$

Note that, vertex B and D will satisfy $x - y + 1 = 0$ and $7x - y - 5 = 0$, respectively.

Since, option (c) satisfy $7x - y - 5 = 0$, therefore coordinate of vertex D is $\left(\frac{1}{3}, \frac{-8}{3}\right)$.

19. ***(d)*** Given equation of circle is

$x^2 + y^2 - 8x - 8y - 4 = 0$, whose centre is $C(4, 4)$ and radius

$$= \sqrt{4^2 + 4^2 + 4} = \sqrt{36} = 6$$

Let the centre of required circle be $C_1(x, y)$. Now, as it touch the X-axis, therefore its radius $= |y|$. Also, it touch the circle $x^2 + y^2 - 8x - 8y - 4 = 0$, therefore $CC_1 = 6 + |y|$

$\Rightarrow$ $\sqrt{(x-4)^2 + (y-4)^2} = 6 + |y|$

$\Rightarrow$ $x^2 + 16 - 8x + y^2 + 16 - 8y = 36 + y^2 + 12|y|$

$\Rightarrow$ $x^2 - 8x - 8y + 32 = 36 + 12|y|$

$\Rightarrow$ $x^2 - 8x - 8y - 4 = 12|y|$

Case I If $y > 0$, then we have

$$x^2 - 8x - 8y - 4 = 12y$$

$\Rightarrow$ $x^2 - 8x - 20y - 4 = 0$

$\Rightarrow$ $x^2 - 8x - 4 = 20y$

$\Rightarrow$ $(x - 4)^2 - 20 = 20y$

$\Rightarrow (x - 4)^2 = 20(y + 1)$, which is a parabola

Case II If $y < 0$, then we have

$$x^2 - 8x - 8y - 4 = -12y$$

$\Rightarrow$ $x^2 - 8x - 8y - 4 + 12y = 0$

$\Rightarrow$ $x^2 - 8x + 4y - 4 = 0$

$\Rightarrow$ $x^2 - 8x - 4 = -4y$

$\Rightarrow$ $(x - 4)^2 = 20 - 4y$

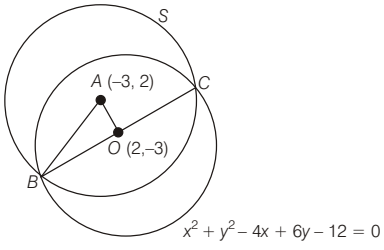
$\Rightarrow (x - 4)^2 = -4(y - 5)$, which is again a parabola.

20. **(b)** Given equation of circle is $x^2 + y^2 - 4x + 6y - 12 = 0$, whose centre is $(2, -3)$ and radius

$$= \sqrt{2^2 + (-3)^2 + 12}$$

$$= \sqrt{4 + 9 + 12} = 5$$

Now, according to given information, we have the adjoining figure.



Clearly, $AO \perp BC$, as O is mid-point of the chord.
Now, in ΔAOB, we have

$$OA = \sqrt{(-3-2)^2 + (2+3)^2}$$

$$= \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

and $OB = 5$

$\therefore$ $AB = \sqrt{OA^2 + OB^2}$

$$= \sqrt{50 + 25} = \sqrt{75} = 5\sqrt{3}$$

21. **(a)** Centre of circle $x^2 + (y + 6)^2 = 1$ is $C(0, -6)$.
Let the coordinates of point P be $(2t^2, 4t)$.
Now, let $D = CP = \sqrt{(2t^2)^2 + (4t + 6)^2}$

$\Rightarrow$ $D = \sqrt{4t^4 + 16t^2 + 36 + 48t}$

$\Rightarrow$ $D^2(t) = 4t^4 + 16t^2 + 48t + 36$

Let $F(t) = 4t^4 + 16t^2 + 48t + 36$

For minimum, $F'(t) = 0$

$\Rightarrow$ $16t^3 + 32t + 48 = 0$

$\Rightarrow$ $t^3 + 2t + 3 = 0$

$\Rightarrow$ $(t + 1)(t^2 - t + 3) = 0.$

$\Rightarrow$ $t = -1$

Thus, coordinate of point P are $(2, -4)$.

Now, $CP = \sqrt{2^2 + (-4 + 6)^2} = \sqrt{4 + 4}$

$= 2\sqrt{2}$

Hence, required equation of circle is

$$(x - 2)^2 + (y + 4)^2 = (2\sqrt{2})^2$$

$\Rightarrow x^2 + 4 - 4x + y^2 + 16 + 8y = 8$

$\Rightarrow$ $x^2 + y^2 - 4x + 8y + 12 = 0$

22. **(c)** Here, we have,

$$\frac{2b^2}{a} = 8 \text{ and } 2b = ae$$

$\Rightarrow$ $b^2 = 4a$ and $2b = ae$

Consider, $2b = ae$

$\Rightarrow$ $4b^2 = a^2e^2$

$\Rightarrow$ $4a^2(e^2 - 1) = a^2e^2$

$\Rightarrow$ $4e^2 - 4 = e^2$ $[\because a \neq 0]$

$\Rightarrow$ $3e^2 = 4$

$\Rightarrow$ $e = \frac{2}{\sqrt{3}}$ $[\because e > 0]$

23. **(b)** Equation of line passing through $(1, -5, 9)$ and parallel to $x = y = z$ is

$$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda \text{ (say)}$$

Thus, any point on this line is of the form $(\lambda + 1, \lambda - 5, \lambda + 9)$.
Now, if $P(\lambda + 1, \lambda - 5, \lambda + 9)$ is the point of intersection of line and plane, then

$$(\lambda + 1) - (\lambda - 5) + \lambda + 9 = 5$$

$\Rightarrow$ $\lambda + 15 = 5$

$\Rightarrow$ $\lambda = -10$

$\Rightarrow$ Coordinates of point P are $(-9, -15, -1)$.
Hence, required distance

$$= \sqrt{(1+9)^2 + (-5+15)^2 + (9+1)^2}$$

$$= \sqrt{10^2 + 10^2 + 10^2}$$

$$= 10\sqrt{3}$$

24. **(d)** Since, the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane $lx + my - z = 9$, therefore we have

$2l - m - 3 = 0$

$[\because$ normal will be perpendicular to the line$]$

$\Rightarrow$ $2l - m = 3$ ...(i)

and $3l - 2m + 4 = 9$

$[\because$ point $(3, -2, -4)$ lies on the plane$]$

$\Rightarrow$ $3l - 2m = 5$ ...(ii)

On solving Eqs. (i) and (ii), we get

$l = 1$ and $m = -1$

$\therefore$ $l^2 + m^2 = 2$

25. **(d)** Given, $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$

and $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\sqrt{3}}{2}(\mathbf{b} + \mathbf{c})$

Now, consider $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\sqrt{3}}{2}(\mathbf{b} + \mathbf{c})$

$\Rightarrow$ $(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = \frac{\sqrt{3}}{2}\mathbf{b} + \frac{\sqrt{3}}{2}\mathbf{c}$

On comparing, we get

$$\mathbf{a} \cdot \mathbf{b} = -\frac{\sqrt{3}}{2}$$

$\Rightarrow$ $|\mathbf{a}||\mathbf{b}| \cos\theta = -\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\cos\theta = -\frac{\sqrt{3}}{2}$ $[\because |\mathbf{a}| = |\mathbf{b}| = 1]$

$\Rightarrow$ $\cos\theta = \cos\left(\pi - \frac{\pi}{6}\right)$

$\Rightarrow$ $\theta = \frac{5\pi}{6}$

26. ***(b)*** We know that, if $x_1, x_2, ..., x_n$ are n observations, then their standard deviation is given by

$$\sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{\Sigma x_i}{n}\right)^2}$$

We have,

$$(3.5)^2 = \frac{(2^2 + 3^2 + a^2 + 11^2)}{4} - \left(\frac{2 + 3 + a + 11}{4}\right)^2$$

$$\Rightarrow \quad \frac{49}{4} = \frac{4 + 9 + a^2 + 121}{4} - \left(\frac{16 + a}{4}\right)^2$$

$$\Rightarrow \quad \frac{49}{4} = \frac{134 + a^2}{4} - \frac{256 + a^2 + 32a}{16}$$

$$\Rightarrow \quad \frac{49}{4} = \frac{4a^2 + 536 - 256 - a^2 - 32a}{16}$$

$$\Rightarrow \quad 49 \times 4 = 3a^2 - 32a + 280$$

$$\Rightarrow \quad 3a^2 - 32a + 84 = 0$$

27. ***(d)*** Clearly, $E_1 = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$

$$E_2 = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)\}$$

and $E_3 = \{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),$
$(3,2),(3,4),(3,6),(4,1),(4,3),(4,5),$
$(5,2),(5,4),(5,6),(6,1),(6,3),(6,5)\}$

$$\Rightarrow \quad P(E_1) = \frac{6}{36} = \frac{1}{6}$$

$$P(E_2) = \frac{6}{36} = \frac{1}{6}$$

and $$P(E_3) = \frac{18}{36} = \frac{1}{2}$$

Now, $P(E_1 \cap E_2) = P$ (getting 4 on die A and 2 on die B)

$$= \frac{1}{36} = P(E_1) \cdot P(E_2)$$

$P(E_2 \cap E_3) = P$ [getting 2 on die B and sum of numbers on both dice is odd]

$$= \frac{3}{36} = P(E_2) \cdot P(E_3)$$

$P(E_1 \cap E_3) = P$ [getting 4 on die A and sum of numbers on both dice is odd]

$$= \frac{3}{36} = P(E_1) \cdot P(E_3)$$

and $P(E_1 \cap E_2 \cap E_3) = P$

[getting 4 on die A, 2 on die B and sum of numbers is odd]

$= P$ (impossible event) $= 0$

Hence, E_1, E_2 and E_3 are not independent.

28. ***(c)*** Given equation is

$$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$$

$$\Rightarrow \quad (\cos x + \cos 3x) + (\cos 2x + \cos 4x) = 0$$

$$\Rightarrow \quad 2\cos 2x \cos x + 2\cos 3x \cos x = 0$$

$$\Rightarrow \quad 2\cos x(\cos 2x + \cos 3x) = 0$$

$$\Rightarrow \quad 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right) = 0$$

$$\Rightarrow \quad \cos x \cdot \cos\frac{5x}{2} \cdot \cos\frac{x}{2} = 0$$

$$\Rightarrow \quad \cos x = 0 \text{ or } \cos\frac{5x}{2} = 0 \text{ or } \cos\frac{x}{2} = 0$$

Now, $\cos x = 0$

$$\Rightarrow \quad x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad [\because 0 \le x < 2\pi]$$

$$\cos\frac{5x}{2} = 0$$

$$\Rightarrow \quad \frac{5x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, ...,$$

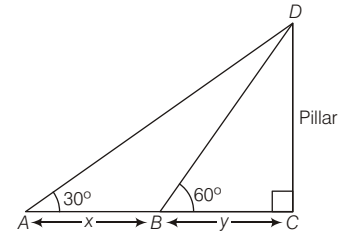
$$\Rightarrow \quad x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5} \qquad [\because 0 \le x < 2\pi]$$

and $\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$

$$\Rightarrow \quad x = \pi \qquad [\because 0 \le x < 2\pi]$$

Hence, $x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

29. ***(d)*** According to given information, we have the following figure



Now, from ΔACD and ΔBCD, we have

$$\tan 30° = \frac{h}{x + y}$$

and $$\tan 60° = \frac{h}{y}$$

$$\Rightarrow \quad h = \frac{x + y}{\sqrt{3}} \qquad ...(i)$$

and $$h = \sqrt{3}\, y \qquad ...(ii)$$

From Eqs. (i) and (ii),

$$\frac{x + y}{\sqrt{3}} = \sqrt{3}\, y$$

$$\Rightarrow \quad x + y = 3y$$

$$\Rightarrow \quad x - 2y = 0$$

$$\Rightarrow \quad y = \frac{x}{2}$$

$\because$ Speed is uniform.

$\therefore$ Distance y will be cover in 5 min.

[$\because$ distance x covered in 10 min

$\therefore$ distance $\frac{x}{2}$ will be cover in 5 min]

30. ***(c)*** Consider, $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$$\equiv [(p \wedge \sim q) \vee q] \vee (\sim p \wedge q)$$
$$\equiv [(p \vee q) \wedge (\sim q \vee q)] \vee (\sim p \wedge q)$$
$$\equiv [(p \vee q) \wedge t] \vee (\sim p \wedge q)$$
$$\equiv (p \vee q) \vee (\sim p \wedge q)$$
$$\equiv (p \vee q \vee \sim p) \wedge (p \vee q \vee q)$$
$$\equiv (q \vee t) \wedge (p \vee q)$$
$$\equiv t \wedge (p \vee q)$$
$$\equiv p \vee q$$

Jitender Gupta

PERIODICITY OF FUNCTIONS

A collection of chapterwise best questions of their types

Only One Option Correct Type

Directions (Q. Nos. 1-15) This section contains 15 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The period of $f(x) = x - [x]$, where $[\cdot]$ denotes greatest integer function, is

(a) π (b) 2
(c) 1 (d) None of these

2. The period of $f(x) = \sin^2 x$ is

(a) $\frac{\pi}{2}$ (b) π
(c) 2π (d) None of these

3. The period of $f(x) = \sin^{-1}(\sin x)$ is

(a) π (b) 2π
(c) $\frac{\pi}{2}$ (d) None of these

4. The period of $f(x) = |\cos x|$ is

(a) $\frac{\pi}{2}$ (b) π
(c) $\frac{3\pi}{2}$ (d) 2π

5. The period of $f(x) = |\cos 2x|$ is

(a) $\frac{\pi}{2}$ (b) π
(c) 2π (d) None of these

6. The period of $f(x) = \tan 4x$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) $\frac{3\pi}{2}$

7. The period of $f(x) = \tan(3x + 2)$ is

(a) π (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$ (d) None of these

8. The period of $f(x) = \sin\left(\frac{x}{3}\right) + \cos\left(\frac{x}{2}\right)$ is

(a) 2π (b) 3π
(c) 6π (d) 12π

9. The period of $f(x) = |\sin x| + |\cos x|$ is

(a) $\frac{\pi}{2}$ (b) π
(c) 2π (d) None of these

10. The period of $f(x) = \sin^4 x + \cos^4 x$ is

(a) π (b) $\frac{\pi}{2}$
(c) 2π (d) None of these

11. The period of $f(x) = \sec(\sin x)$ is

(a) $\frac{\pi}{2}$ (b) 2π
(c) π (d) None of these

12. If $f(x) = \sin\sqrt{[a]}x$, where $[\cdot]$ denotes the greatest integer function, has π as its period, then

(a) $a = 1$ (b) $a = 9$
(c) $a \in [1, 2)$ (d) $a \in [4, 5)$

13. The fundamental period of

$f(x) = x + a - [x + b] + \sin \pi x + \cos 2\pi x + \cos 2n\pi x + \sin 3\pi x + \cos 4\pi x + \ldots + \sin(2n-1)\pi x$

for every $a, b \in R$ (where, $[\cdot]$ denotes greatest integer function), is

(a) 2 (b) 4 (c) 1 (d) 3

14. The period of $f(x) = \sin\frac{\pi}{4}[x] + \cos\frac{\pi x}{2} + \cos\frac{\pi}{3}[x]$, where $[\cdot]$ denotes greatest integer function, is

(a) 8 (b) 12
(c) 24 (d) None of these

15. The period of function

$f(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3x + 15$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) non-periodic

One or More than One Options Correct Type

Directions (Q. Nos. 16-20) This section contains 5 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which one or more than one is/are correct.

16. The period of $f(x) = \sin^4 3x + \cos^4 3x$ is

(a) $\pi/6$ (b) $\pi/3$
(c) $\pi/2$ (d) $\pi/12$

17. The functions which are not periodic, are
(a) $y = [x + 1]$
(b) $y = \sin(x^2)$
(c) $y = \sin^2 x$
(d) $y = \sin^{-1} x$

18. If $f : R \to [-1, 1]$, where $f(x) = \sin\frac{\pi}{2}[x]$ (where, $[\cdot]$ denotes greatest integer function), then
(a) $f(x)$ is even (b) $f(x)$ is odd
(c) $f(x)$ is periodic (d) None of these

19. Identify the statement(s) which is/are incorrect?
(a) The function $f(x) = \cos(\cos^{-1} x)$ is neither odd nor even
(b) The fundamental period of $f(x) = \cos(\sin x) + \cos(\cos x)$ is π
(c) The range of the function $f(x) = \cos(3\sin x)$ is $[-1, 1]$
(d) None of the above

20. If $f(x) = \frac{\sin \pi [x]}{\{x\}}$, then $f(x)$ is
(a) periodic with fundamental period 1
(b) even
(c) range is singleton
(d) None of the above

Match the Column Type

Directions (Q. No. 21) Choices for the correct combination of elements from Column I and Column II are given as options (a), (b), (c) and (d), out of which one is correct.

21. Match the Column I and Column II and mark the correct option from the options given below.

	Column I		Column II
A.	The period of function $f(x) = \sin\left(2\pi t + \frac{\pi}{3}\right) + 2\sin\left(3\pi t + \frac{\pi}{4}\right) + 3\sin 5\pi t$ is	p.	2
B.	The fundamental period of the function $f(x) = \frac{1}{2}\left(\frac{\lvert\sin(\pi/4)x\rvert}{\cos(\pi/4)x} + \frac{\sin(\pi/4)x}{\lvert\cos(\pi/4)x\rvert}\right)$ is	q.	π
C.	The period of $f(x) = \tan\frac{\pi}{2}[x]$ is	r.	3
D.	The period of $f(x) = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$ is	s.	8

Codes

	A	B	C	D
(a)	p	s	p	q
(b)	q	q,r	p	s
(c)	q	s	r,s	p
(d)	s	p	q	r

Linked Comprehension Type

Passage (Q. Nos. 22-24) Given a function

$$f_1(x) = e^{\tan\left\{\frac{x}{4}\right\}} + \cos\pi\left(\frac{(1-2[x])}{2}\right) + \sin\left(\frac{\pi[x]}{2}\right),$$

whose fundamental period is p, (where, $[\cdot]$ and $\{\cdot\}$ represent greatest integer function and fractional part function respectively) and $y = \sqrt{2p + \frac{p}{2}[x] - [x]^2}$. The domain of y is $[q, r)$, one another function $f_2(x) = \begin{cases} 2 + x, & x \geq 0 \\ 2 - x, & x < 0 \end{cases}$.

22. The period p of $f_1(x)$ is
(a) an irrational number
(b) a prime number
(c) a composite number
(d) neither a prime nor a composite number

23. The value of $(r - q - 1)$ is
(a) 6
(b) 7
(c) 8
(d) 9

24. The range of $f_2[f_2(x)]$ in terms of p, q and r is
(a) $[p, \infty)$
(b) $[q, r)$
(c) $[r, \infty)$
(d) $(-\infty, -r) \cup (p, \infty)$

Single Integer Type

Directions (Q. Nos. 25-29) This section contains 5 questions. When worked out will result in an integer from 0 to 9 (both inclusive).

25. The period of $f(x) = \sin\left(\frac{\pi}{2}[x]\right)$ (where, $[\cdot]$ denotes greatest integer function), is

26. The fundamental period of
$$f(x) = e^{\cos^4 \pi x + x - [x] + \cos \pi x}$$
(where, $[\cdot]$ denotes greatest integer function), is

27. The period of $f(x)$ satisfying
$$f(x - 1) + f(x + 3) = f(x + 1) + f(x + 5), \text{ is}$$

28. The period of $f(x)$ satisfying
$f(x + p) = 1 + \{1 - 3f(x) + 3f^2(x) - f^3(x)\}^{1/3}$ is (λp). Then, the value of λ is

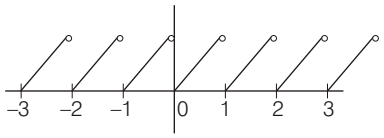
29. The period of $f(x)$ satisfying
$f(x + a) = 1/2 + \sqrt{f(x) - (f(x))^2}$ is (λa). Then, the value of λ is

Answers *with* Explanation

1. ***(c)*** We have, $f(x) = x - [x]$

$$f(x) = \begin{cases} x+2; & -2 \le x < -1 \\ x+1; & -1 \le x < 0 \\ x; & 0 \le x < 1 \\ x-1; & 1 \le x < 2 \\ x-2; & 2 \le x < 3 \end{cases}$$

On sketching the graph,



We see that graph of the function repeats same trend at constant interval 1.

Hence, its period is 1.

Alternate Method

We know that, $f(x + T) = f(x)$

$$f(x + n) = x + n - [x + n]; n \in I$$

$$f(x + n) = x + n - [x] - n = x - [x] = f(x)$$

Least positive value of n is 1.

Hence, its period is 1.

2. ***(b)*** Given, $f(x) = \sin^2 x$

$$\Rightarrow \quad f(x) = \frac{1 - \cos 2x}{2}$$

$$\therefore \quad \text{Period of } f(x) = \frac{2\pi}{2} = \pi$$

[period of $f(ax) = \frac{T}{|a|}$, where T is the period of $f(x)$]

3. ***(b)*** We have, $f(x) = \sin^{-1}(\sin x)$

We know that, if $f(x)$ is non-periodic and $g(x)$ is periodic, then $fog\,(x)$ is periodic with period of $g(x)$ but gof is not periodic.

Hence, $f(x) = \sin^{-1}(\sin x)$ is periodic and

its period = period of $\sin x = 2\pi$

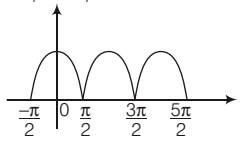
4. ***(b)*** We have, $f(x) = |\cos x|$

$$\Rightarrow \quad f(x) = \sqrt{\cos^2 x} = \sqrt{\frac{1 + \cos 2x}{2}}$$

$$\therefore \quad \text{Period} = \frac{2\pi}{2} = \pi$$

Alternate Method

From the graph of $|\cos x|$, we can see that its period is π.



5. ***(a)*** We have, $f(x) = |\cos 2x|$

We know that, period of $|\cos x|$ is π.

$\therefore$ Period of $|\cos 2x| = \frac{\pi}{2}$

6. ***(a)*** We have, $f(x) = \tan 4x$

If a function transform from $f(x)$ to $f(ax + b)$, then period of the function changes $|T|$ to $\left|\frac{T}{a}\right|$.

We know that, period of $\tan x$ is π.

So, period of $\tan 4x$ is $\frac{\pi}{4}$.

7. ***(c)*** We have, $f(x) = \tan(3x + 2)$

So, its period is $\frac{\pi}{3}$.

8. ***(d)*** We have, $f(x) = \sin\left(\frac{x}{3}\right) + \cos\left(\frac{x}{2}\right)$

Let $\quad f_1(x) = \sin\left(\frac{x}{3}\right)$ and $f_2(x) = \cos\left(\frac{x}{2}\right)$

$$T_1 = \text{Period of } f_1(x) = \frac{2\pi}{\frac{1}{3}} = 6\pi$$

$$T_2 = \text{Period of } f_2(x) = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$\Rightarrow$ Period of $f(x)$ = LCM of T_1 and T_2

So, period = 12π

9. ***(a)*** We have, $f(x) = |\sin x| + |\cos x|$

Let $\quad T_1 =$ Period of $|\sin x| = \pi$

and $\quad T_2 =$ Period of $|\cos x| = \pi$

Since, $f(x)$ is even function and both are complementary to each other.

$\therefore$ Its period is $\frac{1}{2}$ (LCM of T_1 and T_2)

$$= \frac{1}{2}\pi = \frac{\pi}{2}$$

Note Period of $a|\sin x| + b|\cos x| = \pi$, if $a \neq b$

$$= \frac{\pi}{2}, \text{ if } a = b$$

10. ***(b)*** We have, $f(x) = \sin^4 x + \cos^4 x$

$$f(x) = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$$

$$= 1 - \frac{1}{2}(2\sin x \cdot \cos x)^2$$

$$= 1 - \frac{1}{2}\sin^2 2x$$

$$= 1 - \frac{1}{2}\left(\frac{1 - \cos 4x}{2}\right)$$

$$= 1 - \frac{1}{4}(1 - \cos 4x)$$

$\therefore$ Its period = $\frac{2\pi}{4} = \frac{\pi}{2}$

11. ***(c)*** We have, $f(x) = \sec(\sin x)$, $f(x + \pi) = \sec(\sin(x + \pi))$

$$= \sec(-\sin x)$$

$$= \sec(\sin x) = f(x)$$

Hence, its period is π.

12. ***(d)*** We have, $f(x) = \sin(\sqrt{[a]}x)$

Its period $= \frac{2\pi}{\sqrt{[a]}} = \pi$ [given]

$\Rightarrow \quad \sqrt{[a]} = 2$

$\Rightarrow \quad [a] = 4$

$\Rightarrow \quad 4 \le a < 5$

$\therefore \quad a \in [4, 5)$

13. ***(a)*** We have,

$f(x) = (x + a) - [x + b] + \sin\pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + \ldots + \sin(2n - 1)\pi x + \cos 2n\pi x$

To period of $x + a - [x + b] + b - b$

$\Rightarrow \quad x + b - [x + b] + a - b$

$\Rightarrow \quad \{x + b\} + a - b$

Hence, we see that its period is 1.

Now, period of $\sin\pi x$ is $\frac{2\pi}{\pi} = 2$.

Period of $\cos 2\pi x$ is $\frac{2\pi}{2\pi} = 1$.

Similarly, period of

$$\cos 2n\pi x = \frac{2\pi}{2n\pi} = \frac{1}{n}$$

$\therefore$ Period of $f(x)$ is LCM of all above period, which is 2.

14. ***(c)*** We have, $f(x) = \sin\frac{\pi}{4}[x] + \cos\frac{\pi x}{2} + \cos\frac{\pi}{3}[x]$

Let T_1 = Period of $\sin\frac{\pi}{4}[x]$

$\Rightarrow \quad \sin\frac{\pi}{4}[x + T_1] = \sin\frac{\pi}{4}[x]$

$\Rightarrow \quad \frac{\pi}{4}[x + T_1] = 2n\pi + \frac{\pi}{4}[x]$

$\Rightarrow \quad [x + T_1] - [x] = 8n$

gives the value of T independent of x only, if T is an integer.

Hence, above equation reduces to

$$[x] + T_1 - [x] = 8n$$

$\therefore \quad T_1 = 8n$

Its minimum value is 8.

Hence, $T_1 = 8$

$$T_2 = \text{Period of } \cos\frac{\pi x}{2} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$T_3 = \text{Period of } \cos\frac{\pi}{3}[x] = 6$$

$\therefore$ Period of $[x]$ = LCM of T_1, T_2 and $T_3 = 24$

15. ***(a)*** We have,

$$f(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3x + 15$$

$$= -x + [x] - \left(x + \frac{1}{3}\right) + \left[x + \frac{1}{3}\right] - \left(x + \frac{2}{3}\right) + \left[x + \frac{2}{3}\right] + 15 + \frac{1}{3} + \frac{2}{3}$$

$$\Rightarrow \quad f(x) = -\{x\} - \left\{x + \frac{1}{3}\right\} - \left\{x + \frac{2}{3}\right\} + 16$$

$$\therefore \quad f\left(x + \frac{1}{3}\right) = -\left\{x + \frac{1}{3}\right\} - \left\{x + \frac{2}{3}\right\} - \{x + 1\} + 16$$

$$= -\left\{x + \frac{1}{3}\right\} - \left\{x + \frac{2}{3}\right\} - \{x\} + 16 \text{ as } \{x + 1\} = \{x\}$$

$$\Rightarrow f\left(x + \frac{1}{3}\right) = f(x)$$

$\therefore$ Period of $f(x)$ is $\left(\frac{1}{3}\right)$.

16. ***(a)*** We have, $f(x) = \sin^4 3x + \cos^4 3x$

We know that, period of $\sin^4 x + \cos^4 x$ is $\frac{\pi}{2}$.

$\therefore$ Period of $f(x) = \frac{1}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$

17. ***(a,b,d)*** (a) $y = [x + 1]$ is not periodic because $x + 1$ is not periodic.

(b) $y = \sin(x)^2$ is not periodic because x^2 is not periodic.

(c) $y = \sin^2 x$ is periodic because $\sin x$ is periodic.

(d) $y = \sin^{-1} x$ is not periodic.

18. ***(c)*** We have, $f(x) = \sin\left(\frac{\pi}{2}[x]\right)$

To check $f(x)$ is even or odd.

$$f(-x) = \sin\left(\frac{\pi}{2}[-x]\right)$$

Clearly, we cannot say anything because x is not given integer or not integer.

Hence, $f(x)$ is not even or odd.

To check periodicity,

$$f(x + T) = f(x)$$

$$f(x + T) = \sin\left(\frac{\pi}{2}[x + T]\right) = \sin\left(\frac{\pi}{2}[x]\right)$$

$\Rightarrow \quad \frac{\pi}{2}[x + T] = 2n\pi + \frac{\pi}{2}[x]$

$\Rightarrow \quad [x + T] = 4n + [x]$

$\Rightarrow \quad T + [x] = 4n + [x]$

$\therefore \quad T = 4n$

Hence, $f(x)$ is periodic with period 4.

19. ***(a,b,c)*** (a) $f(x) = \cos(\cos^{-1} x) = x$; $-1 \le x \le 1$, then

$f(-x) = \cos\cos^{-1}(-x) = -x$

$\therefore$ It is odd function.

Hence, it is incorrect.

(b) We have,

$$f(x) = \cos(\sin x) + \cos(\cos x)$$

Let its period be T.

$$f(x + T) = f(x)$$

$$\cos(\sin(x + T)) + \cos(\cos(x + T)) = \cos(\sin x) + \cos(\cos x)$$

Put $x = 0$, we get

$$\cos(\sin T) + \cos(\cos T) = \cos 0 + \cos 1$$

or

$$\cos(\sin T) + \cos(\cos T) = \cos\left(\cos\left(\frac{\pi}{2}\right)\right) + \cos\left(\sin\left(\frac{\pi}{2}\right)\right)$$

By comparing, we get $T = \frac{\pi}{2}$

So, its period is $\frac{\pi}{2}$.

Hence, it is incorrect.

(c) We have, $f(x) = \cos(3\sin x)$

We know that, $-1 \le \cos\theta \le 1$

$\Rightarrow \quad f(x) \in [-1, 1]$

20. ***(b,c)*** We have, $f(x) = \frac{\sin\pi[x]}{\{x\}}$

Let T_1 be the period of $\sin\pi[x]$.

Then, $\sin\pi[T_1 + x] = \sin\pi[x]$

$\Rightarrow \quad \pi[T_1 + x] = 2n\pi + \pi[x]$

$\Rightarrow \quad T_1 + [x] = 2n + [x]$

$\therefore \quad T_1 = 2n$

and minimum value is 2.

$$T_1 = 2$$

T_2 = Period of $\{x\}$ is 1.

Hence, period of $f(x)$ = LCM of (T_1 and T_2)

$= 2$

To find range of $f(x) = \sin\pi[x]$ is always 0

Hence, range of $f(x) = 0$, which is a singleton set.

Since, $f(x)$ is always 0, $\forall x \in R$.

$\therefore f(x)$ is an even function.

21. ***(a)***

A. $f(x) = \sin\left(2\pi t + \frac{\pi}{3}\right) + 2\sin\left(3\pi t + \frac{\pi}{4}\right) + 3\sin(5\pi t)$

Let period of $\sin\left(2\pi t + \frac{\pi}{3}\right) = T_1 = \frac{2\pi}{2\pi} = 1$

Period of $\sin\left(3\pi t + \frac{\pi}{4}\right) = T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$

Period of $3\sin 5\pi t = T_3 = \frac{2\pi}{5\pi} = \frac{2}{5}$

Now, period of $f(x) = \frac{\text{LCM of numerator of } T_1, T_2 \text{ and } T_3}{\text{HCF of denominator of } T_1, T_2 \text{ and } T_3}$

$= \frac{\text{LCM of } (1, 2 \text{and } 2)}{\text{HCF of } (1, 3 \text{ and } 5)} = 2$

Thus, A $\rightarrow$ p

B. We have, $f(x) = \frac{1}{2}\left(\frac{\left|\sin\left(\frac{\pi}{4}\right)x\right|}{\cos\left(\frac{\pi}{4}\right)x} + \frac{\sin\left(\frac{\pi}{4}\right)x}{\left|\cos\left(\frac{\pi}{4}\right)x\right|}\right)$

We know that, period of

$\left|\sin\frac{\pi x}{4}\right|$ and $\left|\cos\frac{\pi x}{4}\right|$ is $\frac{\pi}{\frac{\pi}{4}} = 4$.

and period of $\cos\frac{\pi x}{4}$ and $\sin\frac{\pi x}{4}$ is

$$\frac{2\pi}{\frac{\pi}{4}} = 8$$

Now, period of $\frac{\left|\sin\frac{\pi}{4}x\right|}{\cos\frac{\pi x}{4}}$ is 8.

and period of $\frac{\sin\frac{\pi x}{4}}{\left|\cos\frac{\pi x}{4}\right|}$ is 8.

Hence, period of $f(x)$ is 8.

Thus, B $\rightarrow$ s

C. Let period of $f(x)$ be T.

Then, $\tan\left(\frac{\pi}{2}[x + T]\right) = \tan\frac{\pi}{2}[x]$

$\Rightarrow \quad \frac{\pi}{2}[x + T] = n\pi + \frac{\pi}{2}[x]$

$\Rightarrow \quad [x + T] = 2n + [x]$

$\Rightarrow \quad T + [x] = 2n + [x]$

$\therefore \quad T = 2n$

and minimum +ve value of T is 2 and hence its period is 2.

Thus, C $\rightarrow$ p

D. By inspection, we check π can be period of $f(x)$ or not.

$$f(\pi + x) = \frac{\sin(\pi + x) + \sin 3(\pi + x)}{\cos(\pi + x) + \cos 3(\pi + x)}$$

$$= \frac{-\sin x - \sin 3x}{-\cos x - \cos 3x}$$

$$= \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = f(x)$$

We see that, $f(\pi + x) = f(x)$

Hence, π is the period of $f(x)$.

Thus, D $\rightarrow$ q

Solutions (Q. Nos. 22-24)

We have,

$$f_1(x) = e^{\tan\left\{\frac{x}{4}\right\}} + \cos\pi\left(\frac{1 - 2[x]}{2}\right) + \sin\left(\frac{\pi[x]}{2}\right)$$

Let T_1 = Period of $e^{\tan\left\{\frac{x}{4}\right\}} = \frac{1}{\frac{1}{4}} = 4$

T_2 = Period of $\cos\pi\left(\frac{1 - 2[x]}{2}\right)$

or $\cos\left(\frac{\pi}{2} - \pi[x]\right) = \sin\frac{\pi[x]}{2}$

$\Rightarrow \quad \sin\frac{\pi}{2}[x + T_2] = \sin\frac{\pi}{2}[x]$

$\Rightarrow \quad \frac{\pi}{2}[x + T_2] = 2n\pi + \frac{\pi}{2}[x]$

$\Rightarrow \quad [x + T_2] = 4n + [x] \Rightarrow \quad T_2 = 4n$

$\because$ Its period, $T_2 = 4$

So, period of $f_1(x)$ is $p = 4$.

We have, $y = \sqrt{2p + \frac{p}{2}[x] - [x]^2}$

We have calculated, $p = 4$

$\therefore \quad y = \sqrt{8 + 2[x] - [x]^2}$

Domain $= 8 + 2[x] - [x]^2 \geq 0$

$\Rightarrow \quad [x]^2 - 2[x] - 8 \leq 0$

$\Rightarrow \quad ([x] - 4)([x] + 2) \leq 0$

$\Rightarrow \quad -2 \leq [x] \leq 4$

$\Rightarrow \quad -2 \leq x < 5$

$[n_2 \leq [x] \leq n_1 \Rightarrow n_2 \leq x < n_1 + 1$, where $n, n_2 \in I]$

Since, we have domain of y is $[q, r)$.

Hence, by comparing, we get

$q = -2$ and $r = 5$.

22. ***(c)*** $p = 4$, which is a composite number.

23. ***(a)*** The value of $r - q - 1$

$= 5 + 2 - 1 = 6$

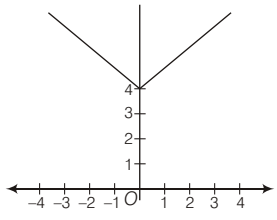
24. ***(a)*** We have, $f_2(x) = \begin{cases} 2 + x; & x \geq 0 \\ 2 - x; & x < 0 \end{cases}$

$$f_2[f_2(x)] = \begin{cases} 2 + f_2(x); & f_2(x) \geq 0 \\ 2 - f_2(x); & f_2(x) < 0 \end{cases}$$

$\Rightarrow \quad f_2[f_2(x)] = 2 + f_2(x) \qquad (\because f_2(x) \geq 2 \ \forall \ x \in R)$

$$= \begin{cases} 2 + 2 + x; & x \geq 0 \\ 2 + 2 - x; & x < 0 \end{cases}$$

$$= \begin{cases} 4 + x; & x \geq 0 \\ 4 - x; & x < 0 \end{cases}$$



Hence, by graph the range of $f_2(x)$ is $[4, \infty)$.

and we know that $p = 4$, hence range is $[p, \infty)$.

25. ***(4)*** Let period of $f(x)$ be T.

$\Rightarrow \quad f(x + T) = f(x)$

$\Rightarrow \quad \sin\left(\frac{\pi}{2}[x + T]\right) = \sin\frac{\pi}{2}[x]$

$\Rightarrow \quad \frac{\pi}{2}[x + T] = 2n\pi + \frac{\pi}{2}[x]$

$\Rightarrow \quad \pi[x + T] = 4n\pi + [x]\pi$

$\Rightarrow \quad [x + T] = 4n + [x]$

$\Rightarrow \quad T + [x] = 4n + [x]$

$\Rightarrow \quad T = 4n$

So, minimum value is 4.

Hence, its period is 4.

26. ***(2)*** We have, $f(x) = e^{\cos^4 \pi x + x - [x] + \cos \pi x}$

Period of $\cos^4 \pi x = \frac{\pi}{\pi} = 1 = T_1$ [say]

Period of $x - [x] = 1 \ = T_2$ [say]

Period of $\cos \pi x = \frac{2\pi}{\pi} = 2 = T_3$ [say]

Now, period of $f(x)$ = LCM of T_1, T_2 and $T_3 = 2$

Hence, its period is 2.

27. ***(8)*** We have, $f(x - 1) + f(x + 3) = f(x + 1) + f(x + 5)$ ...(i)

Replacing x by $x + 2$ in Eq. (i), we get

$f(x + 1) + f(x + 5) = f(x + 3) + f(x + 7)$ ...(ii)

On adding Eqs. (i) and (ii), we get

$f(x - 1) = f(x + 7)$ ...(iii)

Now, replacing x by $x + 1$ in Eq. (iii), we get

$f(x) = f(x + 8)$

Hence, period of $f(x)$ is 8.

28. ***(2)*** We have, $f(x + p) = 1 + \{1 - 3f(x) + 3f^2(x) - f^3(x)\}^{1/3}$

$\Rightarrow \quad f(x + p) = 1 + [\{1 - f(x)\}^3]^{1/3}$

$\Rightarrow \quad f(x + p) = 1 + 1 - f(x)$

$\Rightarrow \quad f(x + p) + f(x) = 2$ ...(i)

Now, replacing x by $x + p$ in Eq. (i), we get

$f(x + 2p) + f(x + p) = 2$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$f(x + 2p) - f(x) = 0$

$\Rightarrow \quad f(x + 2p) = f(x)$

On comparing, we get $\lambda p = 2p$

$\therefore \quad \lambda = 2$

29. ***(2)*** We have, $f(x + a) = \frac{1}{2} + \sqrt{f(x) - (f(x))^2}$ ...(i)

Replacing x by $x + a$ in Eq. (i), we get

$f(x + a + a) = \frac{1}{2} + \sqrt{f(x + a) - (f(x + a))^2}$ ...(ii)

On putting the value of $f(x + a)$ from Eq. (i) in Eq. (ii), we get

$$f(x + 2a) = \frac{1}{2} + \sqrt{\frac{1}{2} + \sqrt{f(x) - (f(x))^2} - \left\{\frac{1}{2} + \sqrt{f(x) - (f(x))^2}\right\}^2}$$

$$\Rightarrow f(x + 2a) = \frac{1}{2} + \sqrt{\frac{1}{2} + \sqrt{f(x) - (f(x))^2} - \frac{1}{4} - f(x) + (f(x))^2 - \sqrt{f(x) - (f(x))^2}}$$

$$\Rightarrow \quad f(x + 2a) = \frac{1}{2} + \sqrt{\frac{1}{2} - \frac{1}{4} - f(x) + (f(x))^2}$$

$$= \frac{1}{2} + \sqrt{\frac{1}{4} - f(x) + (f(x))^2}$$

$$= \frac{1}{2} + \sqrt{\left(\frac{1}{2} - f(x)\right)^2}$$

$$= \frac{1}{2} + f(x) - \frac{1}{2}$$

$\Rightarrow \quad f(x + 2a) = f(x)$

So, its period is $2a$ but it is given λa.

Hence, $\lambda = 2$

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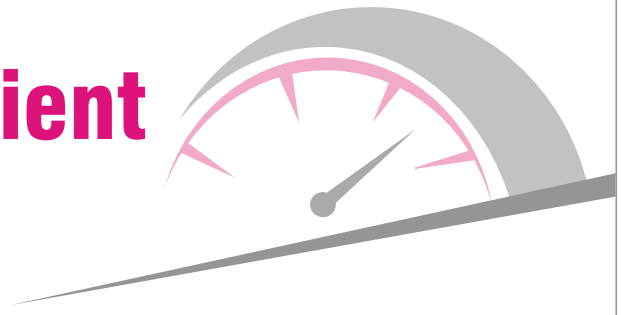
Knowledge Coefficient

Quizzer (No.18)



1. Let cubic $f(x)$ tends to zero at $x = -2$ and has relative maximum/minimum at $x = -1$ and $x = \frac{1}{3}$. If $\int_{-1}^{1} f(x)dx = \frac{14}{3}$, then $f(x)$ is equal to

(a) $x^3 + x^2 + x + 1$ (b) $x^3 + x^2 - x + 1$
(c) $x^3 + x^2 + x + 2$ (d) $x^3 + x^2 - x + 2$

2. Let $f : [2, 7] \to [0, \infty)$ be a continuous and differentiable function. Then, the value of $(f(7) - f(2))\frac{[(f(7))^2 + (f(2))^2 + f(2) \cdot f(7)]}{3}$ is (where, $c \in (2, 7)$)

(a) $3f^2(c)\, f'(c)$
(b) $5f^2(c) \cdot f(c)$
(c) $5f^2(c) \cdot f'(c)$
(d) None of the above

3. The values of x satisfying $\frac{[x]}{[x-2]} - \frac{[x-2]}{[x]} = \frac{8\{x\} + 12}{[x-2][x]}$, where $[.]$ denotes the greatest integer function and $\{\}$ denotes the fractional part function, are

(a) $4, \frac{11}{2}$ (b) $4, \frac{2}{11}$
(c) $\frac{1}{4}, \frac{11}{2}$ (d) $\frac{1}{4}, \frac{2}{11}$

4. Consider three matrices $P = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $R = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$. Then, the value of the sum $\text{tr}(P) + \text{tr}\left(\frac{PQR}{2}\right) + \text{tr}\left(\frac{P(QR)^2}{4}\right) + \text{tr}\left(\frac{P(QR)^3}{8}\right) + \ldots \infty$ is

(a) 6 (b) 9
(c) 12 (d) None of these

5. Let $z = x + iy$ be a complex number, where x and y are integers. Then, the area of the rectangle whose vertices are the roots of the equation $\bar{z}z^3 + z\bar{z}^3 = 350$ is

(a) 48 sq units (b) 32 sq units (c) 40 sq units (d) 80 sq units

6. In a certain test, there are n questions. In the test 2^{n-i} students gave wrong answers to atleast i questions, where $i = 1, 2, 3, \ldots, n$. If the total number of wrong answers given is 2047, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

7. Number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

8. The value of $\sqrt{\pi\left(\int_0^{2016} x\,|\sin \pi x|\, dx\right)}$ is equal to

(a) $\sqrt{2016}$ (b) $\pi\sqrt{2016}$
(c) 1008 (d) 2016

9. If $a_1, a_2, a_3, \ldots$ are in AP such that $a_i \neq 0$, then $\lim_{n \to \infty}\left(\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \ldots + \frac{1}{a_n a_{n+1}}\right)$ is equal to

(a) 0 (b) $\frac{1}{a_1(a_2 + a_1)}$
(c) $\frac{1}{a_1(a_2 - a_1)}$ (d) None of these

10. The intercepts made on the axes by the plane which bisects the line joining the points (1, 2, 3) and (–3, 4, 5) at right angles, are

(a) $\left(-\frac{9}{2}, 9, 9\right)$ (b) $\left(\frac{9}{2}, 9, 9\right)$
(c) $\left(9, -\frac{9}{2}, 9\right)$ (d) $\left(9, \frac{9}{2}, 9\right)$

KNOWLEDGE Coefficient

Quizzer (No. 18)

Q. No.	a.	b.	c.	d.	Q. No.	a.	b.	c.	d.
1.	○	○	○	○	6.	○	○	○	○
2.	○	○	○	○	7.	○	○	○	○
3.	○	○	○	○	8.	○	○	○	○
4.	○	○	○	○	9.	○	○	○	○
5.	○	○	○	○	10.	○	○	○	○

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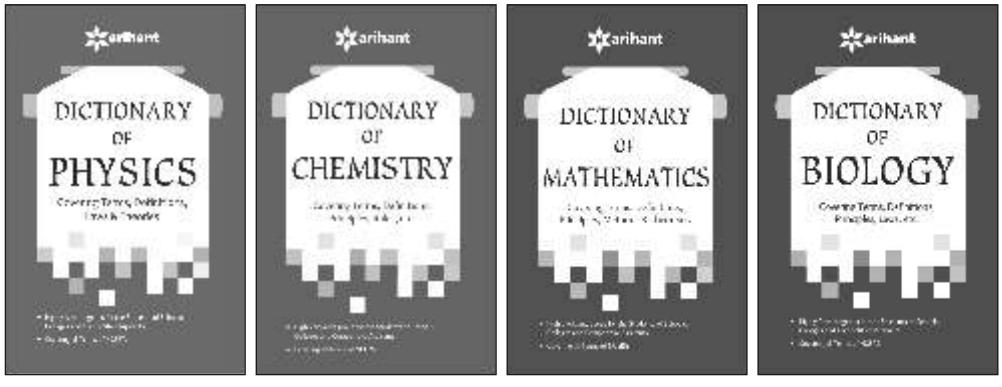

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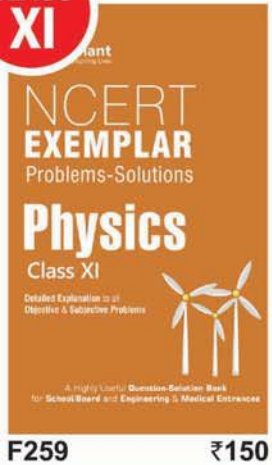

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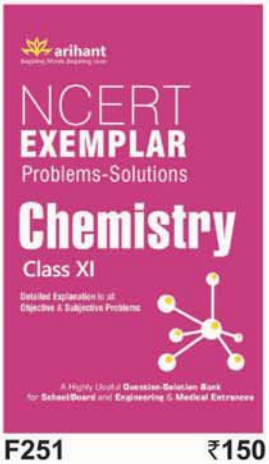

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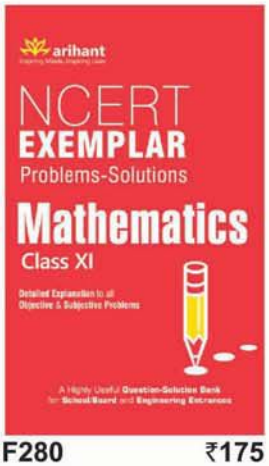

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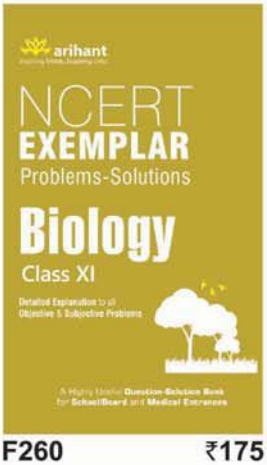

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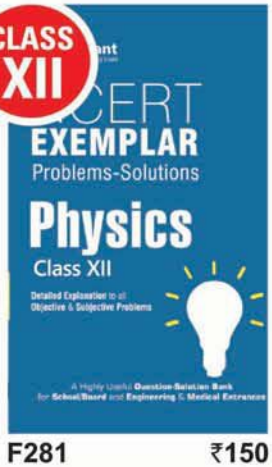

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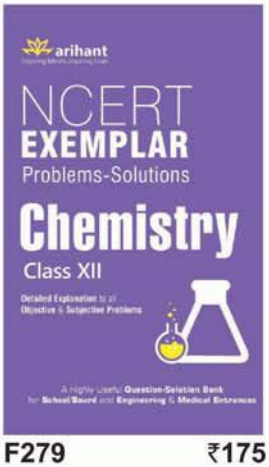

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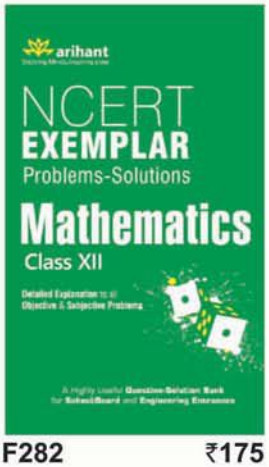

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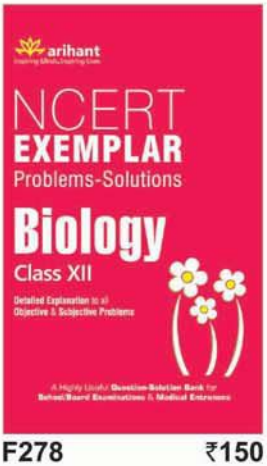

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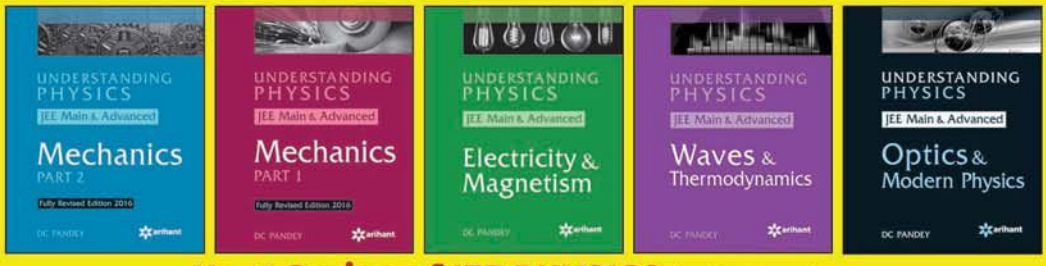

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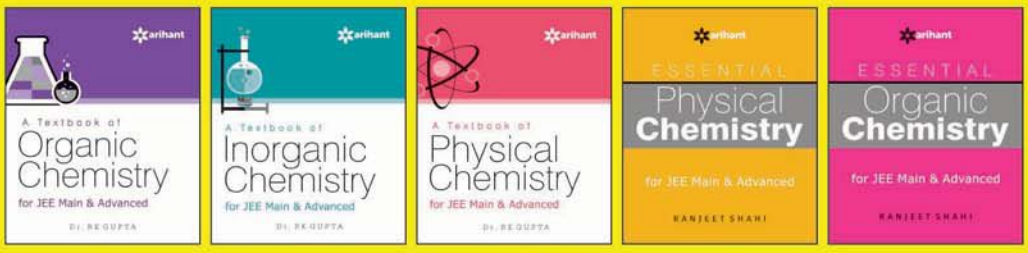

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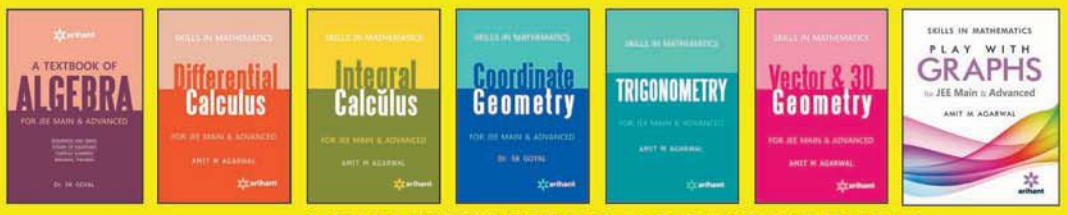

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